Sound Level Meter
- Development of Signal Processing Algorithms

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1 RELEVANT QUANTITIES WITH RELATIONS BETWEEN THEM

1.1 Basic Definitions

Sound intensity[11] is the amount of energy that is transferred through a unit surface perpendicular to the direction of wave propagation, in unit time:

$$w = p^2 / \rho c \ \ [W/m^2], \tag{1.1}$$

where p is the effective pressure, ρ is air density and c is the speed of sound.

Sound level in dB (decibels) is defined as[11]

$$L = 10\log(w/w_0) = 20\log(p/p_0)\ dB\ . \tag{1.2}$$

$w_0 = 10^{-12}\,W/m^2$ is the threshold of human ear (i.e. the smallest sound intensity that is perceived by human ear), p is the effective pressure and $p_0 = 20\mu Pa$ is the effective pressure that corresponds to w_0.

A-**weighted sound pressure level**[11] in dBA:

$$L_A(t) = 10\log\left(\frac{p_A(t)}{p_0}\right)^2 dB\,, \tag{1.3}$$

where $p_A(t)$ is the effective sound pressure, measured by an instrument with frequency weighting[5] A. Similar are the definitions for other frequency weighting (B,C,D).

Remark: Tables of frequency weighting characteristics usually specify relative frequency response in decibels at a given frequency. Let us denote these values by $A(f)[dB]$. Given $A(f)$, we would like to establish relationship between p and p_A for a sinusoidal signal of a given frequency f. We have

$$L_A = 10\log(p_A/p_0)^2 = L - A(f) = 10\log(p/p_0)^2 - A(f)$$

This yields

$$0.1A(f) = \log\left((p_A/p_0)^2/(p/p_0)^2\right) = \log(p_A/p)^2$$

and finally

$$p_A{}^2 = p^2 \cdot 10^{0.1A(f)} \tag{1.4}$$

Similarly we could write

$$L_A(t) = 10\log(w_A/w_0), \tag{1.5}$$

and in that case the relation between the flat and weighted sound intensity would be

$$w_A = w \cdot 10^{0.1A(f)} \tag{1.6}$$

Equivalent continuous A-weighted sound pressure level or average A-weighted sound pressure level is defined as

$$L_{Aeq,T} = 10\lg\left[\frac{\frac{1}{T}\int_{t_1}^{t_2} p_A{}^2(t)\,d\,t}{p_0{}^2}\right] dB; \quad T = t_2 - t_1, \tag{1.7}$$

where $p_A(t)$ is the instantaneous A-weighted sound pressure level of the sound signal and p_0 is the reference pressure of $20\,\mu Pa$. Similar definitions apply for frequency weighting characteristics other than A.

A-weighted sound exposure level is defined as

$$L_{EA,T} = 10\lg\left[\frac{\int_{t_1}^{t_2} p_A{}^2(t)\,d\,t}{T_0\, p_0{}^2}\right] dB, \tag{1.8}$$

where $T_0 = 1s$ (one second). The quantity $p_0{}^2 T_0 = 4 \cdot 10^{-10} Pa^2 s$ is the **reference sound exposure** . The integral expression in the above equation is the **A-weighted sound exposure**,

$$E_{AT} = \int_{t_1}^{t_2} p_A{}^2(t)dt \,. \tag{1.9}$$

The equivalent continuous A-weighted sound pressure level $L_{Aeq,T}$ can be alternatively expressed as

$$L_{Aeq,T} = 10\lg\left(\frac{1}{T}\int_{t_1}^{t_2} 10^{0.1L_A(t)}\, dt\right) \tag{1.10}$$

The weighted sound exposure level is related to the equivalent continuous weighted sound pressure level in the following way:

$$L_{EA,T} = L_{Aeq,T} + 10\lg(T/T_0). \tag{1.11}$$

1.1.1 Definition of Some Auxiliary Functions

For use in further text some auxiliary functions are defined herein; The function *db* converts ratio of r.m.s. (root of mean square) values of two signals to decibels:

$$db(R_w) = 10\log_{10}(R_w) = 10\ln(R_w)/\ln(10)\,. \tag{1.12}$$

Its inverse function *invdb* converts the ratio in decibels back to the ratio of the r.m.s. values:

$$invdb(db) = 10^{0.1db}\,. \tag{1.13}$$

The function *dbamp* converts the ratio of amplitudes of two sinusoidal signals to decibels:

$$dbamp(R_a) = 20\log_{10}(R_a). \tag{1.14}$$

Its inverse function *invdbamp* converts the ratio in decibels back to the ratio of amplitudes:

$$invdbamp(db) = 10^{0.05db}. \tag{1.15}$$

The frequency is often expressed in terms of the standard frequency index *n*. The function *fstd* is used to convert this frequency index to the frequency:

$$fstd(n) = 1kHz \cdot 10^{n/10} \tag{1.16}$$

Its inverse function calculates the frequency index that corresponds to a given frequency:

$$invfstd(f) = 10\log_{10}(f/1kHz). \tag{1.17}$$

The responses in standards are usually specified for the frequencies whose frequency index is an integer. These frequencies are however stated by some nominal numbers rounded to specific number of digits. If we want to evaluate the exact frequencies for which responses are specified, we must therefore find the nearest frequency to the nominal one for which the frequency number is an integer. Responses for different types of frequency weighting are specified in standards for frequencies with integer frequency indices from –20 (corresponding to 10 Hz exactly) to 13 (corresponding to approximately 19950 Hz or nominally 20kHz).

1.2 Requirements Regarding the Quantities to Be Measured

According to international standards [5] and [8] and according to [10] the instrument should be capable of measuring $L_A(t)$, $L_{Aeq,t}$ and $L_{EA,T}$ for different frequency (A, C, flat, optionally B, D) and time weighting characteristics (slow, fast impulse, peak, 10ms). In general, only one frequency and time weighting time may apply at a time.

For weighted sound pressure level functions peak hold, max, min and L_x should be included. Meaning of L_x: L_1 for example is the level of noise that is exceeded in 1% of sampling time. [14] includes for example L_1, L_5, L_{10}, L_{50}, L_{90}, L_{95} and L_{99}.

The above quantities are measured also by [13] [14] and.

[12] mentions sampling rate of $1/20\mu s$. [10] states the requirement for two channel measurement of noise in natural and resident environment for $L_{Aeq,T}$ and $L_{AIeq,T}$, because the correction for outstanding tones is calculated from these two quantities. Furthermore, a filter for obtaining results in $\frac{1}{3}$ octaves is necessary because of the correction for outstanding tones[1].

RMS and peak detection usually run in parallel.

I haven't noticed explicitly mentioned digital signal processing (DSP), but this probably is the case with [14] since it mentions (literally) "Calculation sampling $20\,\mu s$.

It seems possible that the B&K meter uses analogue electronics for frequency weighting. This seems possible because the external filters are used for octave and $\frac{1}{3}$ octave analysis. The signal sent to the external filter is frequency weighted.

In the case of [13], only one frequency/time weighting regime can take place at a time. Seven overlapping 70 dB measuring ranges are available, i.e. 60-130, 50-120, 40-110, 30-100, 20-90, 10-80 and 0-70 dB.

[1] It is not yet clear to me if all bands should operate at the same time.

2 FILTERS

2.1 Analogue Low Pass and High Pass Filters

2.1.1 Low Pass Filter

An analogue version of the low pass filter is shown in Figure 2.1. We see that

$$U_i = U_R + U_0 = R\,I_R + V_0 = R\,C\frac{d\,U_0}{d\,t} + U_0\,.$$

I_R is the electrical current through the resistance (and through the condensor since these currents are equal), *R* is the resistance and *C* is the capacity. The output voltage U_o is therefore in the following relation with the input voltage U_i :

$$R\,C\frac{d\,U_o}{d\,t} + U_o = U_i\,. \tag{2.1}$$

If U_i is a sinusoidal signal, then U_o will also be sinusoidal with different phase and amplitude. We write sinusoidal input and output signal as

$$\begin{aligned} U_i &= A_i e^{i\omega t} \\ U_o &= A_o e^{i\omega t} \end{aligned}\;, \tag{2.2}$$

where the amplitudes A_i and A_o will in general be complex in order to account for phase, and *i* is the imaginary unit. By setting (2.0 into (2.0 we have

$$RC\,A_o\,i\,\omega e^{i\omega t} + A_o\,e^{i\omega t} = A_i\,e^{i\omega t}$$

and therefore

$$\frac{A_o}{A_i} = \frac{1}{1 + RC\,i\,\omega}\,. \tag{2.3}$$

The amplitude response of a low pass filter is therefore

$$\left|\frac{U_0}{U_i}\right| = \frac{1}{\sqrt{1+(RC)^2\omega^2}} = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}} = \frac{\omega_0}{\sqrt{\omega_0^{\;2}+\omega^2}}\,, \tag{2.4}$$

where $\omega_0 = 1/RC$. The phase response ϕ can also be obtained from (2.0. Its tangent $tg(\phi)$ is the ratio between the imaginary and real part of A_o/A_i.

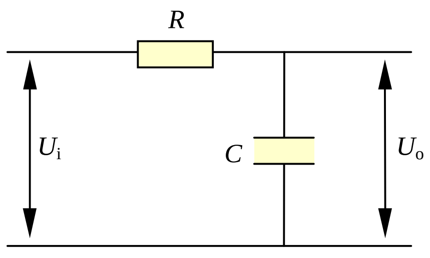

Figure 2.1: Scheme of a passive low pass filter.

2.1.2 High Pass Filter

An analogue version of the high pass filter is shown in Figure 2.2. We see that $U_i = U_C + U_0$ and therefore

$$\frac{dU_i}{dt} = \frac{dU_C}{dt} + \frac{dU_o}{dt} = \frac{I}{C} + \frac{dU_o}{dt} = \frac{U_o}{RC} + \frac{dU_o}{dt}\,.$$

We have taken into account that the current through a condensor equals the current through the resistant. The input and output voltage are so related by the following equation:

$$\frac{dU_i}{dt} = \frac{dU_o}{dt} + \frac{U_o}{RC}\,. \tag{2.5}$$

Remark: The above equations can also obtained from the low pass filter equation by taking into account that

$$U_i = U_{ol} + U_{oh}\,,$$

where U_{ol} is the output voltage of the low pass filter and U_{oh} is the output voltage of the high pass filter with the same *R* and *C*.

By using again (2.0 in (2.0 we obtain the relation between complex input and output amplitudes:

$$i\,\omega\,A_i\,e^{i\omega t} = i\,\omega\,A_o\,e^{i\omega t} + \frac{1}{RC}\,A_o\,e^{i\omega t}$$

This gives

$$\frac{A_o}{A_i} = \frac{i\,\omega}{i\,\omega + 1/RC} = \frac{1}{1 + \dfrac{1}{i\,\omega RC}}\,. \tag{2.6}$$

The amplitude response of the high pass filter is therefore

$$\left|\frac{U_0}{U_i}\right| = \frac{1}{\sqrt{1 + \dfrac{1}{(RC)^2\omega^2}}} = \frac{1}{\sqrt{1 + \left(\dfrac{\omega_0}{\omega}\right)^2}} = \frac{\omega}{\sqrt{\omega^2 + {\omega_0}^2}} \tag{2.7}$$

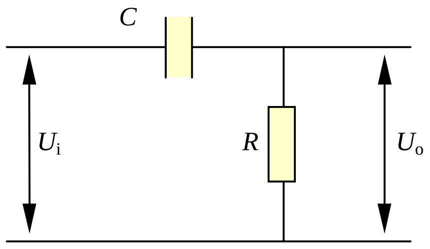

Figure 2.2: Scheme of a passive high pass filter.

2.2 Digital Filters

2.2.1 Digital Low Pass Filter

We will construct a digital filter that works on sampled data and has similar frequency response that the correspondent low pass filter. In order to do that, we take the differential equation that describes the analogue filter and replace derivatives of the quantities with difference of consequent samples of these quantities. With other words, we approximate

$$\frac{dU_o}{dt} \approx \frac{U_o(i)-U_o(i-1)}{t_i - t_{i-1}} = \frac{U_o(i)-U_o(i-1)}{T}, \tag{2.8}$$

where $U_o(i)$ is the output voltage of the *i*- th sample and T is the time period between two consecutive samples (an analogue formula will be used for dU_i/dt). By inserting (2.0 into the low pass filter equation (2.0 we obtain

$$\frac{RC}{T}\left(U_o(i)-U_o(i-1)\right)+U_o(i)=U_i(i)$$

and then with some rearrangement

$$U_o(i) = U_o(i-1) + \frac{T}{T+RC}\left(U_i(i) - U_o(i-1)\right) \tag{2.9}$$

We will write this as

$$\underline{\underline{U_o(i) = aU_i(i) + bU_o(i-1)}}, \tag{2.10}$$

where

$$\begin{aligned} a &= \frac{T}{T+RC} \\ b &= \frac{RC}{T+RC} \end{aligned} \ . \tag{2.11}$$

Again we use the assumption that input and output samples are sinusoidal, which can be written for sampled input and output data as

$$\begin{aligned} U_i(n) &= A_i e^{i\omega nT} \\ U_o(n) &= A_o e^{i\omega nT} \end{aligned} , \tag{2.12}$$

since $t_n = nT$. Inserting this into (2.0 yields

$$\begin{aligned} & A_o\, e^{i\omega nT} = a\, A_i\, e^{i\omega nT} + b\, A_o\, e^{i\omega(n-1)T} = \\ & = a\, A_i\, e^{i\omega nT} + b\, A_o\, e^{-i\omega T}\, e^{i\omega nT} \end{aligned} . \tag{2.13}$$

This gives the formula

$$\frac{A_o}{A_i} = \frac{a}{1 - b\,e^{-i\omega T}} \ , \tag{2.14}$$

from which we obtain the amplitude frequency response of the low pass digital filter:

$$\left|\frac{U_o(i)}{U_i(i)}\right| = \frac{a}{\sqrt{1 + b^2 - 2b\cos(\omega T)}} . \tag{2.15}$$

When $T/RC \to 0$, the above response converges to the response of the analogue low pass filter (2.0.

2.2.2 Digital High Pass Filter

The high pass digital filter will be constructed by inserting approximation (2.0 and the analogue formula for dU_i/dt into (2.0, which yields

$$\frac{U_i(i)-U_i(i-1)}{T}=\frac{U_0(i)-U_0(i-1)}{T}+\frac{U_o(i)}{RC}.$$

Rearrangement yields

$$U_o(i)\frac{T+RC}{T\,RC}=\frac{U_i(i)-U_i(i-1)+U_o(i-1)}{T}$$

and finally

$$\underline{\underline{U_o(i)=c\big(U_i(i)-U_i(i-1)+U_o(i-1)\big)}},\tag{2.16}$$

where

$$\underline{c=\frac{RC}{T+RC}}\tag{2.17}$$

Applying the setting (2.0 in (2.0 yields after some rearrangement

$$A_o=c\left(A_i-A_i\,e^{-i\omega T}+A_o\,e^{-i\omega T}\right),$$

from which follows

$$\frac{A_o}{A_i}=c\frac{1-e^{-i\omega T}}{1-ce^{-i\omega T}}.\tag{2.18}$$

This expression will be further developed:

$$\frac{A_o}{A_i}=c\frac{1-\cos(\omega T)+i\sin(\omega T)}{1-c\cos(\omega T)+c\,i\sin(\omega T)}=c\frac{a_1+i\,b_1}{a_2+i\,b_2},\tag{2.19}$$

where

$$\begin{aligned} a_1 &= 1-\cos(\omega T) \\ b_1 &= \sin(\omega \mathrm{T}) \\ a_2 &= 1-c\cos(\omega T) \\ b_2 &= c\sin(\omega \mathrm{T}) \end{aligned} \quad . \tag{2.20}$$

a_1, $i\,b_1$, a_2 and $i\,b_2$ are real and imaginary parts of the counter and denominator in (2.0. We will further develop this equation by multiplying the counter and denominator by $a_2 - i\,b_2$:

$$\frac{A_o}{A_i} = c\left(\frac{a_1a_2 + b_1b_2}{a_2{}^2 + b_2{}^2} + i\,\frac{a_2b_1 - a_1b_2}{a_2{}^2 + b_2{}^2}\right) \tag{2.21}$$

The amplitude frequency response of the digital high pass filter is so

$$\left|\frac{U_o}{U_i}\right| = c\sqrt{\left(\frac{a_1a_2 + b_1b_2}{a_2{}^2 + b_2{}^2}\right)^2 + \left(\frac{a_2b_1 - a_1b_2}{a_2{}^2 + b_2{}^2}\right)^2}\ , \tag{2.22}$$

where coefficient *c* is defined by (2.0 and coefficients a_1, b_1, a_2 and b_2 are defined by (2.0.

2.3 Frequency Weighting Filters

For sound level meters, frequency must be weighted according to one of the weighting curves A, B or C specified in [5]. These frequency weighting regimes can be achieved by a set of low pass and high pass filters shown in Figure 2.3. Characteristic frequencies f_0 are also shown in the figure, where

$$f_0 = \frac{1}{2\pi}\omega_0 = \frac{1}{2\pi RC}\,, \tag{2.23}$$

where *R* and *C* are the resistance and capacity of filter elements as they appear in equations (2.0 And (2.0 and ω_0 is the corresponding characteristic angular frequency.

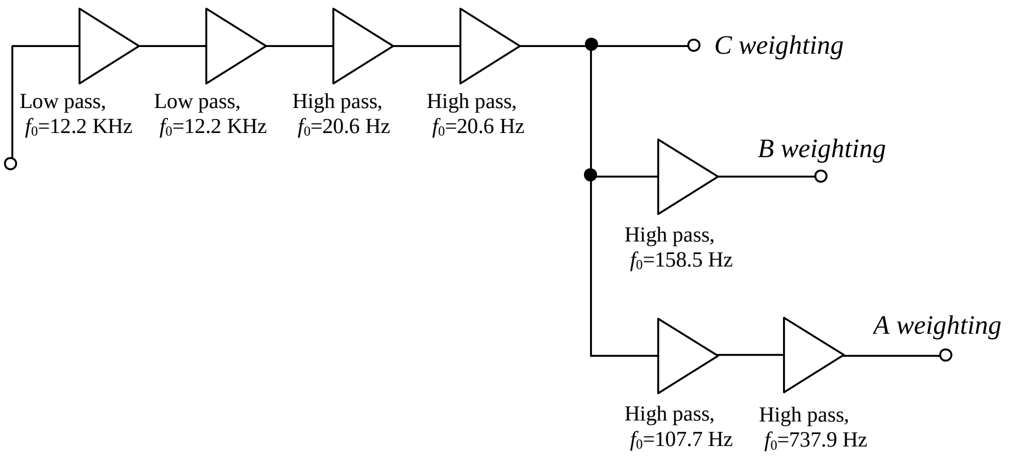

Figure 2.3: Scheme of analogue filter sets which can be used for frequency weighting. Characteristic frequencies are shown for individual high or low pass filters.

Theoretical response of filters in Figure 2.3 is obtained by multiplying individual responses of individual low pass (2.0 and high pass filters (2.0. In this way we obtain

$$R_A(f)\left(=\frac{\overline{|U_{oA}|}}{\overline{|U_i|}}\right)= \frac{(12200Hz)^2 f^4}{\left(f^2+(20.6Hz)^2\right)\left(f^2+(12200Hz)^2\right)\sqrt{f^2+(107.7Hz)^2}\sqrt{f^2+(737.9Hz)^2}} \tag{2.24}$$

$$R_B(f)\left(=\frac{\overline{|U_{oB}|}}{\overline{|U_i|}}\right)= \frac{(12200Hz)^2 f^3}{\left(f^2+(20.6Hz)^2\right)\left(f^2+(12200Hz)^2\right)\sqrt{f^2+(158.5Hz)^2}} \tag{2.25}$$

$$R_C(f)\left(=\frac{\overline{|U_{oC}|}}{\overline{|U_i|}}\right)= \\ \frac{(12200Hz)^2 f^2}{\left(f^2+(20.6Hz)^2\right)\left(f^2+(12200Hz)^2\right)} \tag{2.26}$$

If filters are combined as shown in Figure 2.3, the theoretical responses will differ from those required by the standard for approximately a constant factor at all frequencies specified in the standard. The response of weighting filters must be corrected for the following constants (in decibels) in order to match the requirements of the standard:

$$\begin{aligned} Cor_A &= 1.9998\,dB \\ Cor_B &= 0.169603\,dB \\ Cor_C &= 0.061847\,dB \end{aligned} \tag{2.27}$$

Equivalently, input of filters must be multiplied by the following amplitude factors:

$$\begin{aligned} Coramp_A &= 1.2589 \\ Coramp_B &= 1.01972 \\ Coramp_C &= 1.007146 \end{aligned} \tag{2.28}$$

2.4 Digital Frequency Weighting Filters

Since components of digital filters will have non-uniform properties, to eliminate noise, etc., the frequency weighting filters will be implemented in digital form (Figure 2.4). Weighting filters are in this case implemented as a combination of digital low and high pass filters (equations (2.0 and (2.0), which is a digital equivalent of the filters shown in Figure 2.3.

What concerns algorithms, several digital filters are combined in a filter set in such a way that the input signal is first processed by the first filter, its output is processed by the second filter, output of that filter is processed by the third filter, etc.

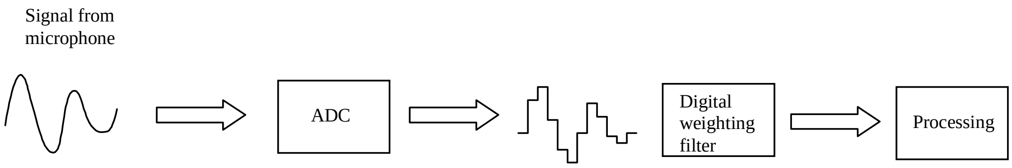

Figure 2.4: Processing of the signal from microphone.

2.4.1.1 Algorithm for Implementing Digital Frequency Weighting Filters

A digital low pass or high pass filter can be described by the following equation

$$O_i = \sum_{j=0}^{M} a_j I_{i-j} + \sum_{k=1}^{N} b_j O_{i-k} \ . \tag{2.29}$$

Here O_i denotes the i-th output (filtered) sample and I_i denotes the i-th input sample. An individual sample of the filtered signal is obtained as a combination of the corresponding input sample (factor a_0), a certain number of earlier input samples (factors a_1, a_2, etc.) and a certain number of earlier (already calculated) output samples (factors b_1, b_2, etc.). When a signal is processed by such a filter, M input and N output samples must be stored in two buffers (input and output) of sizes M and N, respectively. After an output sample is evaluated, the two buffers must be shifted right (the last values are dropped), the calculated output sample must be stored in the first position of the second buffer, and the forthcoming input sample is stored in the first position of the first buffer.

According to equations (2.0 and (2.0, the coefficients for a digital low pass filter are

$$\begin{aligned} a_0^l &= \frac{T}{T+RC} = \frac{T}{T + 1/2\pi\, f_o^l} \\ b_1^l &= \frac{RC}{T+RC} = \frac{1/2\pi\, f_o^l}{T + 1/2\pi\, f_o^l} \end{aligned} \tag{2.30}$$

and according to (2.0 and (2.0 the coefficients for a digital high pass coefficients are

$$a_0^h = \frac{RC}{T+RC} = \frac{1/2\pi\, f_0^h}{T + 1/2\pi\, f_0^h}$$
$$a_1^h = -\frac{RC}{T+RC} = -\frac{1/2\pi\, f_0^h}{T + 1/2\pi\, f_0^h} \qquad (2.31)$$
$$b_1^h = \frac{RC}{T+RC} = \frac{1/2\pi\, f_0^h}{T + 1/2\pi\, f_0^h}$$

The digital frequency weighting filters were obtained by combining a series of digital low pass and high pass filter as shown in Figure 2.3. The signal is filtered in such a way that it is first filtered by the first filter in the series, its output is then filtered by the second filter in the series, etc., using equations (2.0 and (2.0 or (2.0 to apply individual filters to a signal, dependent on which filters constitute the filter series (see Figure 2.3).

Let us for example combine filters *1* and *2*, where the first filter is defined by equation

$$O_i^{(1)} = \sum_{j=0}^{M_1} a_j^{(1)} I_{i-j}^{(1)} + \sum_{k=1}^{N_1} b_k^{(1)} O_{i-k}^{(1)} \qquad (2.32)$$

and the second one by equation

$$O_i^{(2)} = \sum_{j=0}^{M_2} a_j^{(2)} I_{i-j}^{(2)} + \sum_{k=1}^{N_2} b_k^{(2)} O_{i-k}^{(2)} \ . \qquad (2.33)$$

For implementation of the combination of these two filters we must store M_1 values of the original signal, $\max(N_1, M_2+1)$ values of the first filter output and N_2 values of the second filter output. Input of the second filter equals output of the first one:

$$I_i^{(2)} = O_i^{(1)}, \qquad (2.34)$$

therefore (2.0 becomes

$$O_i^{(2)} = \sum_{j=0}^{M_2} a_j^{(2)} O_{i-j}^{(1)} + \sum_{k=1}^{N_2} b_k^{(2)} O_{i-k}^{(2)} \ . \qquad (2.35)$$

The C weighting filter is implemented in the following way:

$$\begin{aligned}
&O_i^{l1} = a_o^{l1} I_i^{l1} + b_1^{l1} O_{i-1}^{l1} , \\
&O_i^{(l2)} = a_o^{(l2)} O_i^{(l1)} + b_1^{(l2)} O_{i-1}^{(l2)} , \\
&O_i^{(h3)} = a_o^{(h3)} O_i^{(l2)} + a_1^{(h3)} O_{i-1}^{(l2)} + b_1^{(h3)} O_{i-1}^{(h3)} , \ . \\
&O_i^{(h4)} = a_o^{(h4)} O_i^{(h3)} + a_1^{(h4)} O_{i-1}^{(h3)} + b_1^{(h4)} O_{i-1}^{(h4)} , \\
&i = 3, 4, ...
\end{aligned} \tag{2.36}$$

We must store O_{i-1}^{l1}, O_i^{l1}, O_{i-1}^{l2}, O_i^{l2}, O_{i-1}^{h3}, O_i^{h3} and O_{i-1}^{h4} for the next sample. We apply the above formula to samples $i = 3, 4, ...$ and set

$$\begin{aligned}
&O_1^{(h4)} = I_1 \\
&O_2^{(h4)} = I_2 \\
&O_1^{(h3)} = I_1 \\
&O_2^{(h3)} = I_2 \\
&O_1^{(l2)} = I_1 \\
&O_2^{(l2)} = I_2 \\
&O_1^{(l1)} = I_1 \\
&O_2^{(l1)} = I_2
\end{aligned} . \tag{2.37}$$

The B filter is implemented by (2.0 and in addition

$$\begin{aligned}
&O_i^{(h5)} = a_o^{(h5)} O_i^{(h4)} + a_1^{(h5)} O_{i-1}^{(h4)} + b_1^{(h5)} O_{i-1}^{(h5)} , \\
&i = 3, 4, ...
\end{aligned} , \tag{2.38}$$

Initially we set (2.0 and in addition

$$\begin{aligned}
&O_1^{(h5)} = I_1 \\
&O_2^{(h5)} = I_2
\end{aligned} . \tag{2.39}$$

The A filter is implemented by (2.0 and in addition

$$\begin{aligned}
&O_i^{(h6)} = a_o^{(h6)} O_i^{(h5)} + a_1^{(h6)} O_{i-1}^{(h5)} + b_1^{(h6)} O_{i-1}^{(h6)} , \\
&O_i^{(h7)} = a_o^{(h7)} O_i^{(h6)} + a_1^{(h7)} O_{i-1}^{(h6)} + b_1^{(h7)} O_{i-1}^{(h7)} , \ , \\
&i = 3, 4, ...
\end{aligned} \tag{2.40}$$

Initially we set (20. and in addition

$$\begin{aligned} O_1^{(h6)} &= I_1 \\ O_2^{(h6)} &= I_2 \\ O_1^{(h7)} &= I_1 \\ O_2^{(h7)} &= I_2 \end{aligned}. \tag{2.41}$$

Coefficients in the above equations are obtained from (2.0 for low pass and from (2.0 for high pass filters that constitute the series. The corresponding characteristic frequencies are shown in Figure 2.3. Upper indices denote the individual filter of the set for all quantities used in equations (2.0 to (2.0.

2.4.2 Time Discretisation Errors and Corrections

Because of time discretisation (effectively approximation of derivatives by differences) response of a digital filter will not be exactly the sama as response of its analogue equivalent. If samples can be represented in arbitrary accuracy, then digital filter response will limit to the appropriate analogue response when the sampling period T will approach zero. An objective is to find the slowest sampling rate at which the response of digital weighting filters will match the prescribed response enough accurately. Results of the corresponding tests are shown in Table 2.4, Table 2.5 and Table 2.6 below. It turns that the **longest acceptable sampling period** is $T = 10^{-5} s$. All further discussions will therefore assume the **sampling rate** of $10^5 s^{-1}$ if not stated differently.

Deformation of filter response due to time discretisation can be partially compensated by variation of filter parameters (i.e. characteristic frequencies of individual low and high pass filters in a weighting set). Parameters have been optimised for the sampling rate of $10^5 s^{-1}$. The resulting response of a corrected A weighting digital filter is shown in Table 2.7. The maximum relative error (with respect to admitted tolerance) has been approximately halved, which is quite a lucky circumstance for instrument design. Corrections for filter parameters are listed in Table 2.1. Frequency correction factors are defined in a relative manner such that

$$f_i = f_{0i}\left(1 + Cor_{if}\right), \tag{2.42}$$

where f_I is the corrected charasteristic frequency of the *i*-th filter in a weighting filter set, f_{0i} is the uncorrected characteristic frequency corresponding to the appropriate analogue filter, and Cor_{if} is the corresponding relative correction tabulated in Table 2.1. Amplitude corrections are factors by which filtered signals must be multiplied in order to achieve exact agreement with the standard at the calibration frequency of 1 kHz.

Results shown in the tables below apply for A weighting filters. Tesults for other weighting filters are not stated because errors with B and C weighting are nowhere greater than errors with A weighting (in most casesthey are approximately the same).

Table 2.1: Optimal corrections for digital weighting filter parameters at sampling rate $10^5 s^{-1}$.

Filter	**Weighting**	**Uncorrected char. frequency**	**Optimal correction**
Low pass 1	A, B, C	f_{0l1}=12.2 kHz	Cor_{l1f}=0.308381
Low pass 2	A, B, C	f_{0l2}=12.2 kHz	Cor_{l2f}=0.421121
High pass 3	A, B, C	f_{0h3}=20.6 Hz	Cor_{h3f}=-0.0313729
High pass 4	A, B, C	f_{0h4}=20.6 Hz	Cor_{h4f}=0.0498406
High pass 5	B	f_{0h5}=158.5 Hz	Cor_{h5f}=0.0107605
High pass 6	A	f_{0h6}=107.7 Hz	Cor_{h6f}=0.00377726
High pass 7	A	f_{0h7}=737.9 Hz	Cor_{h7f}=0.0223711
Amplitude correction factors to achieve perfect agreement at 1kHz			
Filter set		**Notation**	**Value**
Amplitude cor. A	A	$Corampdig_A$	1.29495
Amplitude cor. B	B	$Corampdig_B$	1.02703
Amplitude cor. C	C	$Corampdig_C$	1.00915

2.4.2.1 Parameters for frequency weighting at sampling rate 48 kHz

At a lower sampling rate, e.g. 48 kHz, optimal parameters for digital weighting filters will differ from those which apply for the sampling rate of 100 kHz. Parameters which give the response closest to theoretical for this sampling rate, are listed in Table 2.2. Meaning of parameters is as described in the previous section. With this sampling rates, errors are greater in magnitude than with the sampling rate of 100 kHz. Maximum relative errors for type 1 response are 0.382 for C weighting, 0.382 for B weighting, and 0.381 for A weighting. Errors were controlled for up to 19952 Hz. Frequency response of the A weighting filter designed for operation at the sampling rate of 48 kHz is shown in Table 2.3.

Table 2.2: Optimal corrections for digital weighting filter parameters at sampling rate $48000\ s^{-1}$.

Filter	Weighting	Uncorrected char. frequency	Optimal correction
Low pass 1	A, B, C	f_{0l1}=12.2 kHz	Cor_{l1f}= 0.70186
Low pass 2	A, B, C	f_{0l2}=12.2 kHz	Cor_{l2f}= 0.30801
High pass 3	A, B, C	f_{0h3}=20.6 Hz	Cor_{h3f}= -0.06693
High pass 4	A, B, C	f_{0h4}=20.6 Hz	Cor_{h4f}= 0.17758
High pass 5	B	f_{0h5}=158.5 Hz	Cor_{h5f}= 0.02531
High pass 6	A	f_{0h6}=107.7 Hz	Cor_{h6f}= -0.06268
High pass 7	A	f_{0h7}=737.9 Hz	Cor_{h7f}= 0.053132
Amplitude correction factors to achieve perfect agreement at 1kHz			
Filter set		**Notation**	**Value**
Amplitude cor. A	A	$Corampdig_A$	1.33966
Amplitude cor. B	B	$Corampdig_B$	1.03763
Amplitude cor. C	C	$Corampdig_C$	1.01372

Table 2.3: Response of A weighting digital filter with sampling rate 48 kHz. Parameters for the filter are listed in Table 2.2.

```
Corrected digital response:
    f [Hz]  an. r. [db]    num. dig.      n.d.-a.  tol. type 1 (n.d.-a.)/+tol.
*       10     -70.4351     -70.4945   -0.0594076 -1e+30 - 3     5.94076e-32
*  12.5893     -63.3758     -63.3861   -0.0103151 -1e+30 - 3     1.03151e-32
*  15.8489     -56.6927     -56.6391    0.0536045 -1e+30 - 3      0.0178682
*  19.9526     -50.4566     -50.3273     0.129271    -3 - 3      0.0430905
*  25.1189     -44.7072     -44.4981     0.209158    -2 - 2       0.104579
*  31.6228      -39.444     -39.1632     0.280816  -1.5 - 1.5     0.187211
*  39.8107     -34.6341     -34.3003     0.333797  -1.5 - 1.5     0.222531
*  50.1187     -30.2318     -29.8708     0.360978  -1.5 - 1.5     0.240652
*  63.0957     -26.1976      -25.838     0.359574  -1.5 - 1.5     0.239716
*  79.4328     -22.5066     -22.1742     0.332441  -1.5 - 1.5     0.221628
*      100     -19.1452     -18.8585     0.286677    -1 - 1       0.286677
*  125.893     -16.1003     -15.8683     0.231988    -1 - 1       0.231988
*  158.489     -13.3518     -13.1744     0.177379    -1 - 1       0.177379
*  199.526     -10.8715     -10.7416     0.129924    -1 - 1       0.129924
*  251.189     -8.63114     -8.53851    0.0926298    -1 - 1      0.0926298
*  316.228     -6.61163     -6.54636    0.0652768    -1 - 1      0.0652768
*  398.107     -4.80889     -4.76296    0.0459276    -1 - 1      0.0459276
*  501.187     -3.23297     -3.20053    0.0324388    -1 - 1      0.0324388
*  630.957     -1.90067     -1.87852     0.022151    -1 - 1       0.022151
*  794.328    -0.823951    -0.811821    0.0121303    -1 - 1      0.0121303
*     1000            0            0            0    -1 - 1              0
*  1258.93     0.591255     0.573334   -0.0179207    -1 - 1      0.0179207
*  1584.89     0.980885     0.935848   -0.0450362    -1 - 1      0.0450362
*  1995.26      1.20024      1.11279   -0.0874487    -1 - 1      0.0874487
```

```
*  2511.89      1.27114      1.11932    -0.151818    -1 - 1       0.151818
*  3162.28      1.19934     0.952735      -0.2466    -1 - 1         0.2466
*  3981.07     0.970799     0.590769     -0.38003    -1 - 1        0.38003
*  5011.87     0.549976  -0.00491042    -0.554887  -1.5 - 1.5     0.369925
*  6309.57    -0.119329    -0.878113    -0.758784    -2 - 1.5     0.379392
*  7943.28     -1.10806     -2.05773    -0.949666    -3 - 1.5     0.316555
*    10000     -2.48833     -3.52731     -1.03898    -4 - 2       0.259744
*  12589.3     -4.31312     -5.19365    -0.880532    -6 - 3       0.146755
*  15848.9     -6.59725     -6.85565    -0.258406 -1e+30 - 3     2.58406e-31
*  19952.6     -9.31078     -8.16829      1.14249 -1e+30 - 3       0.380832

Correction: 2.53991 db, amp. factor 1.33966 (th. cor=1.2589)

Maximum relative error in decibels: 0.382027


Plotting from 1 to 34, range -70.4945 to 2.17956.

X***                                                                         #
    ****X**                                                                  #
            ***X**                                                           #
                   ***X**                                                    #
5                        ***X**                                              #
6                              ***X*                                         #
(7,-34.3003)                         **X*                                    #
(8,-29.8708)                              **X*                               #
(9,-25.838)                                   +**X*                          #
(10,-22.1742)                                     +**X                       #
(11,-18.8585)                                         +*X                    #
(12,-15.8683)                                             *X                 #
(13,-13.1744)                                                *X              #
(14,-10.7416)                                                   *X           #
(15,-8.53851)                                                     *X         #
(16,-6.54636)                                                       +*X      #
(17,-4.76296)                                                         +X     #
(18,-3.20053)                                                           *X   #
(19,-1.87852)                                                             *X #
(20,-0.811821)                                                              X#
(21,0)                                                                       X
(22,0.573334)                                                                X
(23,0.935848)                                                                #X
(24,1.11279)                                                                 #X
(25,1.11932)                                                                 #X
(26,0.952735)                                                                #X
(27,0.590769)                                                                X+
(28,-0.00491042)                                                             X
(29,-0.878113)                                                              X+
(30,-2.05773)                                                             X*+#
(31,-3.52731)                                                              X+  #
(32,-5.19365)                                                           X*+   #
(33,-6.85565)                                                         X*       #
(34,-8.16829)                                                       +X         #
Plotting from 1 to 34, range -70.4945 to 2.17956.
```

2.4.2.2 Antialiasing Filter

There exists a possibility that analogue signal will be filtered by an antialiasing filter before AD conversion and frequency weighting. Such filter should cut off the frequencies higher than the Nyquist critical frequency, whose power would be otherwise aliased to lower frequencies and would spoil the accuracy of the instrument.

Influence of such a filter consisting of two high pass analogue filters with critical frequencies 40 kHz and 80 kHz has been tested. If such filter was applied, parameters of the frequency weighting filters should be further corrected. Optimal corrections are stored in function setoptaliaspar_40_80 in the accompanying software. It turns that weighted response of the set including the antialiasing filter would be even more accurate than the response without this filter, provided that appropriate corrections are applied. Response is also not too sensitive on characteristic frequencies of the included low pass filters, which is a precondition for using analogue components. It turned that characteristic frequencies could be varied from 28 to 45 and from 70 to 90 kHz, respectively.

Table 2.4: Response of uncorrected A weighting digital filter with sampling period $T=10^5$ s. The uncorrected filter is implemented as a set of digital filter with the same parameters as are shown in Figure 2.4, except that filter response is shifted in such a way that response at 1 kHz corresponds to the response required by the standard. Columns of the table represent frequency in Hz, required (analytical) response in decibels, numerically calculated response of the filter, difference between the numerical and analytical response, admitted negative and positive tolerance for type 1, and relative error, i.e. difference between the calculated and required response at a certain frequency divided by the appropriate tolerance for type 1. Results are shown in a draft graphical form below the table. Time discretisation error is acceptable for type 1.

Filter A uncorrected, Ts=1e-5:

	f [Hz]	an. r. [db]	num. dig.	n.d.-a.	tol. type 1	(n.d.-a.)/+tol.
*	10	-70.4351	-70.2646	0.170584	-1e+30 - 3	0.0568612
*	12.5893	-63.3758	-63.2062	0.169643	-1e+30 - 3	0.0565478
*	15.8489	-56.6927	-56.5246	0.168105	-1e+30 - 3	0.0560349
*	19.9526	-50.4566	-50.2901	0.166528	-3 - 3	0.0555095
*	25.1189	-44.7072	-44.5427	0.16454	-2 - 2	0.0822701
*	31.6228	-39.444	-39.2817	0.162374	-1.5 - 1.5	0.108249
*	39.8107	-34.6341	-34.4739	0.160224	-1.5 - 1.5	0.106816
*	50.1187	-30.2318	-30.0745	0.157296	-1.5 - 1.5	0.104864
*	63.0957	-26.1976	-26.0437	0.153871	-1.5 - 1.5	0.102581
*	79.4328	-22.5066	-22.3567	0.149883	-1.5 - 1.5	0.0999222
*	100	-19.1452	-19.0002	0.144961	-1 - 1	0.144961
*	125.893	-16.1003	-15.9609	0.139461	-1 - 1	0.139461
*	158.489	-13.3518	-13.2188	0.133016	-1 - 1	0.133016
*	199.526	-10.8715	-10.7463	0.125206	-1 - 1	0.125206
*	251.189	-8.63114	-8.51527	0.115873	-1 - 1	0.115873
*	316.228	-6.61163	-6.50773	0.103906	-1 - 1	0.103906
*	398.107	-4.80889	-4.72025	0.0886325	-1 - 1	0.0886325
*	501.187	-3.23297	-3.16347	0.0695074	-1 - 1	0.0695074
*	630.957	-1.90067	-1.8536	0.0470735	-1 - 1	0.0470735
*	794.328	-0.823951	-0.800765	0.0231857	-1 - 1	0.0231857
*	1000	0	0	0	-1 - 1	0
*	1258.93	0.591255	0.570128	-0.0211266	-1 - 1	0.0211266
*	1584.89	0.980885	0.940857	-0.0400279	-1 - 1	0.0400279

*	1995.26	1.20024	1.1437	-0.0565333	-1 - 1	0.0565333
*	2511.89	1.27114	1.19856	-0.0725748	-1 - 1	0.0725748
*	3162.28	1.19934	1.1096	-0.0897378	-1 - 1	0.0897378
*	3981.07	0.970799	0.86098	-0.109819	-1 - 1	0.109819
*	5011.87	0.549976	0.416479	-0.133497	-1.5 - 1.5	0.088998
*	6309.57	-0.119329	-0.278055	-0.158727	-2 - 1.5	0.0793633
*	7943.28	-1.10806	-1.28606	-0.177998	-3 - 1.5	0.0593326
*	10000	-2.48833	-2.66212	-0.173785	-4 - 2	0.0434461
*	12589.3	-4.31312	-4.42895	-0.115826	-6 - 3	0.0193043
*	15848.9	-6.59725	-6.55503	0.0422205	-1e+30 - 3	0.0140735
*	19952.6	-9.31078	-8.94213	0.368652	-1e+30 - 3	0.122884
*	25118.9	-12.3893	-11.4241	0.96514		
*	31622.8	-15.7534	-13.7623	1.99114		
*	39810.7	-19.327	-15.6064	3.72056		
*	50118.7	-23.0468	-16.3971	6.64967		
*	63095.7	-26.8652	-15.0821	11.7831		

Correction: **2.17282 db**, amp. factor 1.28422 (th. cor=1.2589)

Maximum relative error: 0.14496

```
X***                                                                          #
    ****X**                                                                   #
            ***X**                                                            #
                   ***X**                                                     #
5                        ***X**                                               #
6                              ***X*                                          #
(7,-34.4739)                          **X*                                    #
(8,-30.0745)                               **X*                               #
(9,-26.0437)                                   **X*                           #
(10,-22.3567)                                      **X*                       #
(11,-19.0002)                                          **X                    #
(12,-15.9609)                                              *X                 #
(13,-13.2188)                                                 *X              #
(14,-10.7463)                                                    *X           #
(15,-8.51527)                                                      *X         #
(16,-6.50773)                                                        +*X      #
(17,-4.72025)                                                          +X     #
(18,-3.16347)                                                            *X   #
(19,-1.8536)                                                               *X #
(20,-0.800765)                                                               X#
(21,0)                                                                        X
(22,0.570128)                                                                 X
(23,0.940857)                                                                 #X
(24,1.1437)                                                                   #X
(25,1.19856)                                                                  #X
(26,1.1096)                                                                   #X
(27,0.86098)                                                                  #X
(28,0.416479)                                                                 X
(29,-0.278055)                                                               X#
(30,-1.28606)                                                               X #
(31,-2.66212)                                                              X  #
(32,-4.42895)                                                            X*   #
(33,-6.55503)                                                         X*      #
(34,-8.94213)                                                      +X*        #
(35,-11.4241)                                                  ++X*           #
(36,-13.7623)                                               +++X*             #
(37,-15.6064)                                           +++  X*               #
(38,-16.3971)                                       ++++    X                 #
(39,-15.0821)                                   +++          X                #
```

Table 2.5: Response of uncorrected digital A weighting filter at sampling period 10^{-4}s (see explanation for **Table 2.4**). It is evident that at such sampling rate the time discretisation error can not be compensated by variation of filter parameters.

Filter A uncorrected, Ts=1e-4:

	f [Hz]	an. r. [db]	num. dig.	n.d.-a.	tol. type 1	(n.d.-a.)/+tol.
*	10	-70.4351	-68.6102	1.8249	-1e+30 - 3	0.608301
*	12.5893	-63.3758	-61.5616	1.81416	-1e+30 - 3	0.604719

*	15.8489	-56.6927	-54.8924	1.80024	-1e+30 - 3	0.600081
*	19.9526	-50.4566	-48.673	1.78358	-3 - 3	0.594527
*	25.1189	-44.7072	-42.9427	1.7645	-2 - 2	0.882252
*	31.6228	-39.444	-37.7011	1.74297	-1.5 - 1.5	1.16198
*	39.8107	-34.6341	-32.9149	1.71921	-1.5 - 1.5	1.14614
*	50.1187	-30.2318	-28.5408	1.69094	-1.5 - 1.5	1.12729
*	63.0957	-26.1976	-24.5406	1.65698	-1.5 - 1.5	1.10466
*	79.4328	-22.5066	-20.8907	1.61591	-1.5 - 1.5	1.07728
*	100	-19.1452	-17.5784	1.56678	-1 - 1	1.56678
*	125.893	-16.1003	-14.5909	1.50941	-1 - 1	1.50941
*	158.489	-13.3518	-11.9087	1.44312	-1 - 1	1.44312
*	199.526	-10.8715	-9.50774	1.36377	-1 - 1	1.36377
*	251.189	-8.63114	-7.36614	1.265	-1 - 1	1.265
*	316.228	-6.61163	-5.47316	1.13848	-1 - 1	1.13848
*	398.107	-4.80889	-3.83215	0.97674	-1 - 1	0.97674
*	501.187	-3.23297	-2.45636	0.776614	-1 - 1	0.776614
*	630.957	-1.90067	-1.359	0.541673	-1 - 1	0.541673
*	794.328	-0.823951	-0.54419	0.279761	-1 - 1	0.279761
*	1000	0	0	0	-1 - 1	0
*	1258.93	0.591255	0.292104	-0.299151	-1 - 1	0.299151
*	1584.89	0.980885	0.352156	-0.628728	-1 - 1	0.628728
*	1995.26	1.20024	0.197879	-1.00236	-1 - 1	1.00236
*	2511.89	1.27114	-0.146942	-1.41808	-1 - 1	1.41808
*	3162.28	1.19934	-0.626298	-1.82563	-1 - 1	1.82563
*	3981.07	0.970799	-1.10972	-2.08052	-1 - 1	2.08052
*	5011.87	0.549976	-1.34589	-1.89587	-1.5 - 1.5	1.26391
*	6309.57	-0.119329	-0.962672	-0.843343	-2 - 1.5	0.421672
*	7943.28	-1.10806	0.162544	1.27061	-3 - 1.5	0.847072
*	10000	-2.48833	-52.559	-50.0707	-4 - 2	12.5177
*	12589.3	-4.31312	-0.204267	4.10885	-6 - 3	1.36962
*	15848.9	-6.59725	-1.18063	5.41661	-1e+30 - 3	1.80554
*	19952.6	-9.31078	-29.4303	-20.1195	-1e+30 - 3	2.01195e-29
*	25118.9	-12.3893	-1.34264	11.0466		
*	31622.8	-15.7534	0.346022	16.0994		
*	39810.7	-19.327	-10.0345	9.29248		
*	50118.7	-23.0468	-15.3235	7.72332		
*	63095.7	-26.8652	-0.579132	26.2861		

Correction: 3.84908 db, amp. factor 1.55759 (th. cor=1.2589)

Maximum relative error: 12.5177

```
++X***                                                                        #
    ++****X**                                                                 #
            ++***X**                                                          #
                   ++***X**                                                   #
5                        ++***X**                                             #
6                              ++***X*                                        #
(7,-32.9149)                         ++**X*                                   #
(8,-28.5408)                              ++**X*                              #
(9,-24.5406)                                  ++**X*                          #
(10,-20.8907)                                     ++**X*                      #
(11,-17.5784)                                         ++**X                   #
(12,-14.5909)                                             ++*X                #
(13,-11.9087)                                                ++*X             #
(14,-9.50774)                                                   +*X           #
(15,-7.36614)                                                     ++*X        #
(16,-5.47316)                                                       ++*X      #
(17,-3.83215)                                                         ++X     #
(18,-2.45636)                                                           +*X   #
(19,-1.359)                                                               +X  #
(20,-0.54419)                                                                X#
(21,0)                                                                        X
(22,0.292104)                                                                 X
(23,0.352156)                                                                 X+
(24,0.197879)                                                                 X+
(25,-0.146942)                                                               X#+
(26,-0.626298)                                                               X#+
(27,-1.10972)                                                               X #+
(28,-1.34589)                                                               X +
(29,-0.962672)                                                               X#
30                                                ****************************X
31                 X*****************************                          +  #
32                                               ****************************X#
(33,-1.18063)                                                 **************X #
(34,-29.4303)                                X***************    ++          #
```

```
(35,-1.34264)                                               ***************X #
(36,0.346022)                                              +++           ****X
(37,-10.0345)                                          +++      **X*****     #
(38,-15.3235)                                      ++++     X*******         #
(39,-0.579132)                                 +++                    ********X#
```

Table 2.6: Response of uncorrected digital A weighting filter at sampling period $3*10^{-5}$s (see explanation for **Table 2.4**). Although the sampling rate is close to the satisfactory rate of 10^{-5} s^{-1}, at such sampling rate the time discretisation error still can not be compensated enough by variation of filter parameters. Problem caused by deflection of low pass filters at higher frequencies are clearly seen. This sampling rate would give satisfactory results for type 2 instrument under the condition that all other parts of the sound lever meter produce error less than half of those admitted by the standard.

Filter A uncorrected, Ts=3e-5:

	f [Hz]	an. r. [db]	num. dig.	n.d.-a.	tol. type 1	(n.d.-a.)/+tol.
*	10	-70.4351	-69.8766	0.558523	-1e+30 - 3	0.186174
*	12.5893	-63.3758	-62.8206	0.555248	-1e+30 - 3	0.185083
*	15.8489	-56.6927	-56.1415	0.551132	-1e+30 - 3	0.183711
*	19.9526	-50.4566	-49.9103	0.546256	-3 - 3	0.182085
*	25.1189	-44.7072	-44.1669	0.540311	-2 - 2	0.270156
*	31.6228	-39.444	-38.91	0.534013	-1.5 - 1.5	0.356009
*	39.8107	-34.6341	-34.1071	0.526946	-1.5 - 1.5	0.351297
*	50.1187	-30.2318	-29.7135	0.518264	-1.5 - 1.5	0.345509
*	63.0957	-26.1976	-25.6894	0.508181	-1.5 - 1.5	0.338787
*	79.4328	-22.5066	-22.0107	0.49593	-1.5 - 1.5	0.33062
*	100	-19.1452	-18.664	0.48114	-1 - 1	0.48114
*	125.893	-16.1003	-15.6361	0.464205	-1 - 1	0.464205
*	158.489	-13.3518	-12.9076	0.444244	-1 - 1	0.444244
*	199.526	-10.8715	-10.4511	0.420452	-1 - 1	0.420452
*	251.189	-8.63114	-8.24002	0.391126	-1 - 1	0.391126
*	316.228	-6.61163	-6.25834	0.353297	-1 - 1	0.353297
*	398.107	-4.80889	-4.50444	0.304447	-1 - 1	0.304447
*	501.187	-3.23297	-2.99008	0.242897	-1 - 1	0.242897
*	630.957	-1.90067	-1.73095	0.16972	-1 - 1	0.16972
*	794.328	-0.823951	-0.7367	0.087251	-1 - 1	0.087251
*	1000	0	0	0	-1 - 1	0
*	1258.93	0.591255	0.498921	-0.0923335	-1 - 1	0.0923335
*	1584.89	0.980885	0.786221	-0.194663	-1 - 1	0.194663
*	1995.26	1.20024	0.883631	-0.316607	-1 - 1	0.316607
*	2511.89	1.27114	0.797521	-0.473616	-1 - 1	0.473616
*	3162.28	1.19934	0.516259	-0.683076	-1 - 1	0.683076
*	3981.07	0.970799	0.0119923	-0.958806	-1 - 1	0.958806
*	5011.87	0.549976	-0.749276	-1.29925	-1.5 - 1.5	0.866168
*	6309.57	-0.119329	-1.78571	-1.66638	-2 - 1.5	0.833192
*	7943.28	-1.10806	-3.06806	-1.96	-3 - 1.5	0.653332
*	10000	-2.48833	-4.47966	-1.99133	-4 - 2	0.497833
*	12589.3	-4.31312	-5.77697	-1.46385	-6 - 3	0.243975
*	15848.9	-6.59725	-6.53455	0.0626964	-1e+30 - 3	0.0208988
*	19952.6	-9.31078	-6.05297	3.25781	-1e+30 - 3	1.08594
*	25118.9	-12.3893	-3.26984	9.11943		
*	31622.8	-15.7534	0.838683	16.5921		
*	39810.7	-19.327	-1.92101	17.406		
*	50118.7	-23.0468	-6.56587	16.4809		

*	63095.7	-26.8652	0.279952	27.1452

Correction: 2.56563 db, amp. factor 1.34364 (th. cor=1.2589)

Maximum relative error in decibels: **1.08594**

```
+X**                                                                        #
    +***X***                                                                #
            ****X**                                                         #
                   ***X**                                                   #
5                        +***X*                                             #
6                             +**X**                                        #
(7,-34.1071)                        ***X*                                   #
(8,-29.7135)                             **X*                               #
(9,-25.6894)                                 +**X*                          #
(10,-22.0107)                                    +**X                       #
(11,-18.664)                                         +*X*                   #
(12,-15.6361)                                            **X                #
(13,-12.9076)                                               +*X             #
(14,-10.4511)                                                  *X           #
(15,-8.24002)                                                    +*X        #
(16,-6.25834)                                                      +*X      #
(17,-4.50444)                                                        +*X    #
(18,-2.99008)                                                          +X   #
(19,-1.73095)                                                            *X #
(20,-0.7367)                                                               X#
(21,0)                                                                      X
(22,0.498921)                                                               X
(23,0.786221)                                                               X+
(24,0.883631)                                                               #X
(25,0.797521)                                                               X+
(26,0.516259)                                                               X+
(27,0.0119923)                                                              X+
(28,-0.749276)                                                             X+
(29,-1.78571)                                                             X+#
(30,-3.06806)                                                           X*+ #
(31,-4.47966)                                                          X +  #
(32,-5.77697)                                                        X*++   #
(33,-6.53455)                                                       +X      #
(34,-6.05297)                                                    ++  X      #
(35,-3.26984)                                                +++       *X*  #
(36,0.838683)                                             +++              **X
(37,-1.92101)                                         +++               **X*#
(38,-6.56587)                                     ++++              X***    #
(39,0.279952)                                 +++                       ****X
```

Table 2.7: Response of **corrected digital A weighting filter** at sampling period $\mathbf{10^{-5}s}$ (see explanation for **Table 2.4**). Significant improvement with respect to uncorrected response shown in **Table 2.4** is achieved. **Maximum relative error 0.078** permits that also for type 1 instruments major source of error are elsewhere. Difference with respect to analogue filter is notable at higher frequences, for which tolerances are not prescribed and should therefore not affect compliance with the standard.

Filter A corrected, Ts=1e-5:

	f [Hz]	an. r. [db]	num. dig.	n.d.-a.	tol. type 1	(n.d.-a.)/+tol.
*	10	-70.4351	-70.5392	-0.104099	-1e+30 - 3	1.04099e-31
*	12.5893	-63.3758	-63.4701	-0.094313	-1e+30 - 3	9.4313e-32
*	15.8489	-56.6927	-56.7747	-0.0820821	-1e+30 - 3	8.20821e-32
*	19.9526	-50.4566	-50.524	-0.067383	-3 - 3	0.022461
*	25.1189	-44.7072	-44.7591	-0.0518727	-2 - 2	0.0259364
*	31.6228	-39.444	-39.481	-0.0369501	-1.5 - 1.5	0.0246334
*	39.8107	-34.6341	-34.6578	-0.0237028	-1.5 - 1.5	0.0158018

*	50.1187	-30.2318	-30.2452	-0.0134629	-1.5 - 1.5	0.00897529
*	63.0957	-26.1976	-26.2033	-0.00575954	-1.5 - 1.5	0.0038397
*	79.4328	-22.5066	-22.5067	-0.000113519	-1.5 - 1.5	7.56793e-05
*	100	-19.1452	-19.1415	0.00367393	-1 - 1	0.00367393
*	125.893	-16.1003	-16.0939	0.0063976	-1 - 1	0.0063976
*	158.489	-13.3518	-13.3437	0.0080958	-1 - 1	0.0080958
*	199.526	-10.8715	-10.8626	0.00886421	-1 - 1	0.00886421
*	251.189	-8.63114	-8.62185	0.0092911	-1 - 1	0.0092911
*	316.228	-6.61163	-6.60247	0.00916445	-1 - 1	0.00916445
*	398.107	-4.80889	-4.80029	0.00859998	-1 - 1	0.00859998
*	501.187	-3.23297	-3.2256	0.00737752	-1 - 1	0.00737752
*	630.957	-1.90067	-1.89525	0.00542718	-1 - 1	0.00542718
*	794.328	-0.823951	-0.820972	0.0029788	-1 - 1	0.0029788
*	1000	0	0	0	-1 - 1	0
*	1258.93	0.591255	0.587137	-0.00411799	-1 - 1	0.00411799
*	1584.89	0.980885	0.970497	-0.0103875	-1 - 1	0.0103875
*	1995.26	1.20024	1.18123	-0.0190086	-1 - 1	0.0190086
*	2511.89	1.27114	1.23924	-0.0318934	-1 - 1	0.0318934
*	3162.28	1.19934	1.14872	-0.0506175	-1 - 1	0.0506175
*	3981.07	0.970799	0.893564	-0.0772344	-1 - 1	0.0772344
*	5011.87	0.549976	0.437041	-0.112935	-1.5 - 1.5	0.07529
*	6309.57	-0.119329	-0.275483	-0.156154	-2 - 1.5	0.078077
*	7943.28	-1.10806	-1.30737	-0.199305	-3 - 1.5	0.066435
*	10000	-2.48833	-2.71193	-0.223598	-4 - 2	0.0558996
*	12589.3	-4.31312	-4.50934	-0.196222	-6 - 3	0.0327037
*	15848.9	-6.59725	-6.66485	-0.0676055	-1e+30 - 3	6.76055e-32
*	19952.6	-9.31078	-9.07744	0.233345	-1e+30 - 3	0.0777818
*	25118.9	-12.3893	-11.5794	0.809827		
*	31622.8	-15.7534	-13.9319	1.82155		
*	39810.7	-19.327	-15.7848	3.54214		
*	50118.7	-23.0468	-16.5788	6.468		
*	63095.7	-26.8652	-15.2582	11.607		

Correction: 2.24505 db, amp. factor 1.29495 (th. cor=1.2589)

Maximum relative error: 0.078077

```
X***                                                                         #
    ****X**                                                                  #
            ***X**                                                           #
                   ***X**                                                    #
5                        ***X**                                              #
6                              ***X*                                         #
(7,-34.6578)                         **X*                                    #
(8,-30.2452)                              **X*                               #
(9,-26.2033)                                  **X*                           #
(10,-22.5067)                                     **X*                       #
(11,-19.1415)                                         **X                    #
(12,-16.0939)                                             *X                 #
(13,-13.3437)                                                *X              #
(14,-10.8626)                                                   *X           #
(15,-8.62185)                                                     *X         #
(16,-6.60247)                                                       *X       #
(17,-4.80029)                                                         *X     #
(18,-3.2256)                                                            *X   #
(19,-1.89525)                                                             *X #
(20,-0.820972)                                                              X#
(21,0)                                                                       X
(22,0.587137)                                                                X
(23,0.970497)                                                                #X
(24,1.18123)                                                                 #X
(25,1.23924)                                                                 #X
(26,1.14872)                                                                 #X
(27,0.893564)                                                                #X
(28,0.437041)                                                                X
(29,-0.275483)                                                              X+
(30,-1.30737)                                                             X+#
(31,-2.71193)                                                            X  #
(32,-4.50934)                                                          X*   #
(33,-6.66485)                                                       X*      #
(34,-9.07744)                                                    +X*        #
(35,-11.5794)                                                 +X*           #
(36,-13.9319)                                             ++X*              #
(37,-15.7848)                                         ++++X*                #
(38,-16.5788)                                     ++++    X                 #
(39,-15.2582)                                 +++          X                #
Plotting from 1 to 39, range -70.5392 to 2.18013.
```

2.4.3 Errors Due To Level Discretisation at ADC Conversion

Individual samples of the signal will be represented by a finite number of bits (**level discretisation**), which will cause additional distortion of digital filter response. Errors caused by level discretisation have been investigated by checking filter response on sinusoidal signals with different resolutions. Amplitude of testing signals was always the same as ADC range, but number of bits used for representation of different levels was varied. Results of numerical investigation are shown in Table 2.9 through Table 2.14.

Results show that **amplitude of a sinusoidal signal** must be **at least 11 bits** in order to obtain response within tolerances. The accuracy at 11 bits would be rather poor for type 1 instruments and satisfactory for type 2. 11 bits means 10^{10}=1024 distinctive positive and negative levels (one bit is a sign bit).

According to the results, 12 bits should be taken as an absolute minimum for amplitude of the lowest signal that can occur during measurement within a certain range if this signal is sinusoidal. At A certain reserve should be taken in practice, at least 3 bits are suggested. Table 2.14 clearly shows how accuracy of frequency weighting is destroyed when the detected signal has too low amplitude.

The total amplitude range of the ADC must be a certain number of bits above the minimum signal amplitude that can be accurately measured. This number depends on level range we would like to have. Each additional bit extends the level range by 6.02 dB:

$$bit \equiv 6.02\,dB\,. \tag{2.43}$$

For example, by 7 additional bits (above the minimum 12, say) we have level range of 40 *dB*, by 10 additional bits we obtain the range of 60 dB and by 14 additional bits the range of 80 dB (Table 2.8). By 16 bit conversion we could cover the range of 20 dB in the best possible case, which is probably out of question. By 24 bit conversion we could in the best case cover the range of 70 dB. In this respect there is not much difference between types 1 and 2, since by lower required accuracy for type 2 at most a single bit can be gained.

Table 2.8: relation between the number of additional bits in level representation and the extension of the level range.

Bits	/ dB
1	6.0206
2	12.0412
3	18.0618
4	24.0824
5	30.103
6	36.1236
7	42.1442
8	48.1648
9	54.1854
10	60.206
11	66.2266
12	72.2472
13	78.2678
14	84.2884
15	90.309
16	96.3296
17	102.35
18	108.371
19	114.391
20	120.412

Table 2.9: Response of **corrected digital A weighting filter** at sampling period **10^{-5}s** with limited (**8 bits**) signal representation (see explanation for **Table 2.4**). Problems occur at low frequencies where the signal has an expressive step form, which causes deviations in the high pass filter response. Response is obtained with sinusoidal signal of amplitude equal to the ADC range (i.e. the highest level of the signal which is representable).

Corrected digital A filter, Ts=1e-5 s, limited representation (8 bits)

	f [Hz]	an. r. [db]	num. dig.	n.d.-a.	tol. type 1	(n.d.-a.)/+tol.
*	10	-70.4351	-51.2148	19.2204	-1e+30 - 3	6.40678
*	12.5893	-63.3758	-51.6205	11.7553	-1e+30 - 3	3.91843
*	15.8489	-56.6927	-51.2532	5.43942	-1e+30 - 3	1.81314
*	19.9526	-50.4566	-48.7553	1.70132	-3 - 3	0.567107
*	25.1189	-44.7072	-44.3066	0.400583	-2 - 2	0.200292
*	31.6228	-39.444	-39.3598	0.0841926	-1.5 - 1.5	0.0561284
*	39.8107	-34.6341	-34.618	0.0160588	-1.5 - 1.5	0.0107059
*	50.1187	-30.2318	-30.2275	0.00430965	-1.5 - 1.5	0.0028731
*	63.0957	-26.1976	-26.1907	0.00690297	-1.5 - 1.5	0.00460198
*	79.4328	-22.5066	-22.4969	0.00974059	-1.5 - 1.5	0.00649373
*	100	-19.1452	-19.1323	0.0128889	-1 - 1	0.0128889
*	125.893	-16.1003	-16.0847	0.0155984	-1 - 1	0.0155984
*	158.489	-13.3518	-13.3352	0.0165983	-1 - 1	0.0165983
*	199.526	-10.8715	-10.8541	0.0174383	-1 - 1	0.0174383
*	251.189	-8.63114	-8.61323	0.0179079	-1 - 1	0.0179079
*	316.228	-6.61163	-6.59412	0.0175098	-1 - 1	0.0175098
*	398.107	-4.80889	-4.79187	0.0170147	-1 - 1	0.0170147
*	501.187	-3.23297	-3.21713	0.0158427	-1 - 1	0.0158427
*	630.957	-1.90067	-1.88692	0.0137497	-1 - 1	0.0137497
*	794.328	-0.823951	-0.812513	0.0114385	-1 - 1	0.0114385
*	1000	0	0	0	-1 - 1	0
*	1258.93	0.591255	0.595523	0.00426843	-1 - 1	0.00426843
*	1584.89	0.980885	0.978947	-0.00193742	-1 - 1	0.00193742
*	1995.26	1.20024	1.18959	-0.0106521	-1 - 1	0.0106521

*	2511.89	1.27114	1.2477	-0.0234344	-1 - 1	0.0234344
*	3162.28	1.19934	1.15728	-0.0420544	-1 - 1	0.0420544
*	3981.07	0.970799	0.902015	-0.0687835	-1 - 1	0.0687835
*	5011.87	0.549976	0.445454	-0.104523	-1.5 - 1.5	0.0696819
*	6309.57	-0.119329	-0.267003	-0.147674	-2 - 1.5	0.0738369
*	7943.28	-1.10806	-1.2989	-0.190837	-3 - 1.5	0.0636124
*	10000	-2.48833	-2.70874	-0.220402	-4 - 2	0.0551006
*	12589.3	-4.31312	-4.50082	-0.187697	-6 - 3	0.0312828
*	15848.9	-6.59725	-6.65628	-0.0590299	-1e+30 - 3	5.90299e-32
*	19952.6	-9.31078	-9.06884	0.241949	-1e+30 - 3	0.0806498

Correction: 2.25284 db, amp. factor 1.29611 (th. cor=1.2589)

Maximum relative error: 6.40678

```
++++                 X                                                        #
    +++++++         X                                                         #
            ++++++   X                                                        #
                   ++++*X*                                                    #
5                        +**X**                                               #
6                              ***X*                                          #
(7,-34.618)                          **X*                                     #
(8,-30.2275)                              **X*                                #
(9,-26.1907)                                  **X*                            #
(10,-22.4969)                                     **X*                        #
(11,-19.1323)                                         **X                     #
(12,-16.0847)                                             *X                  #
(13,-13.3352)                                                *X               #
(14,-10.8541)                                                   *X            #
(15,-8.61323)                                                     *X          #
(16,-6.59412)                                                       *X        #
(17,-4.79187)                                                         *X      #
(18,-3.21713)                                                           *X    #
(19,-1.88692)                                                             *X  #
(20,-0.812513)                                                               X#
(21,0)                                                                        X
(22,0.595523)                                                                 X
(23,0.978947)                                                                 #X
(24,1.18959)                                                                  #X
(25,1.2477)                                                                   #X
(26,1.15728)                                                                  #X
(27,0.902015)                                                                 #X
(28,0.445454)                                                                 X
(29,-0.267003)                                                               X#
(30,-1.2989)                                                                X #
(31,-2.70874)                                                              X  #
(32,-4.50082)                                                            X*   #
(33,-6.65628)                                                         X*      #
(34,-9.06884)                                                     +X*         #
```

Table 2.10: Response of corrected digital A weighting filter at sampling period 10^{-5}s with limited (**9 bits**) signal representation (see explanation of **Table 2.9**).

Corrected digital A filter, Ts=1e-5 s, limited representation (9 bits):

	f [Hz]	an. r. [db]	num. dig.	n.d.-a.	tol. type 1	(n.d.-a.)/+tol.
*	10	-70.4351	-59.1758	11.2593	-1e+30 - 3	3.75311
*	12.5893	-63.3758	-58.649	4.72678	-1e+30 - 3	1.57559
*	15.8489	-56.6927	-55.4301	1.26258	-1e+30 - 3	0.420859
*	19.9526	-50.4566	-50.2163	0.240265	-3 - 3	0.0800882
*	25.1189	-44.7072	-44.689	0.0182623	-2 - 2	0.00913116
*	31.6228	-39.444	-39.4587	-0.0146995	-1.5 - 1.5	0.0097997
*	39.8107	-34.6341	-34.6527	-0.0186262	-1.5 - 1.5	0.0124175
*	50.1187	-30.2318	-30.2437	-0.0118978	-1.5 - 1.5	0.00793185
*	63.0957	-26.1976	-26.2022	-0.00461535	-1.5 - 1.5	0.0030769
*	79.4328	-22.5066	-22.5067	-0.000109635	-1.5 - 1.5	7.30902e-05
*	100	-19.1452	-19.1422	0.00298145	-1 - 1	0.00298145

	f [Hz]	an. r. [db]	num. dig.	n.d.-a.	tol. type 1	(n.d.-a.)/+tol.
*	125.893	-16.1003	-16.0942	0.00610025	-1 - 1	0.00610025
*	158.489	-13.3518	-13.3441	0.00769881	-1 - 1	0.00769881
*	199.526	-10.8715	-10.8631	0.00839151	-1 - 1	0.00839151
*	251.189	-8.63114	-8.62239	0.00874622	-1 - 1	0.00874622
*	316.228	-6.61163	-6.60281	0.00882278	-1 - 1	0.00882278
*	398.107	-4.80889	-4.80086	0.00802652	-1 - 1	0.00802652
*	501.187	-3.23297	-3.22604	0.00693662	-1 - 1	0.00693662
*	630.957	-1.90067	-1.89591	0.00476548	-1 - 1	0.00476548
*	794.328	-0.823951	-0.821493	0.00245791	-1 - 1	0.00245791
*	1000	0	0	0	-1 - 1	0
*	1258.93	0.591255	0.586695	-0.00455951	-1 - 1	0.00455951
*	1584.89	0.980885	0.969992	-0.0108924	-1 - 1	0.0108924
*	1995.26	1.20024	1.18078	-0.0194558	-1 - 1	0.0194558
*	2511.89	1.27114	1.23873	-0.0324026	-1 - 1	0.0324026
*	3162.28	1.19934	1.14819	-0.0511411	-1 - 1	0.0511411
*	3981.07	0.970799	0.893008	-0.0777908	-1 - 1	0.0777908
*	5011.87	0.549976	0.436509	-0.113468	-1.5 - 1.5	0.075645
*	6309.57	-0.119329	-0.275956	-0.156627	-2 - 1.5	0.0783135
*	7943.28	-1.10806	-1.30786	-0.199792	-3 - 1.5	0.0665973
*	10000	-2.48833	-2.71347	-0.225138	-4 - 2	0.0562846
*	12589.3	-4.31312	-4.50989	-0.196768	-6 - 3	0.0327946
*	15848.9	-6.59725	-6.66536	-0.0681158	-1e+30 - 3	6.81158e-32
*	19952.6	-9.31078	-9.07794	0.232841	-1e+30 - 3	0.0776136

Correction: 2.2443 db, amp. factor 1.29484 (th. cor=1.2589)

Maximum relative error in decibels: 3.75311

Table 2.11: Response of corrected digital A weighting filter at sampling period 10^{-5}s with limited (**10 bits**) signal representation (see explanation of **Table 2.9**). Error is not acceptable, neither for a type 1 nor for type 2 instrument.

Corrected digital A filter, Ts=1e-5 s, limited representation (10 bits):

	f [Hz]	an. r. [db]	num. dig.	n.d.-a.	tol. type 1	(n.d.-a.)/+tol.
*	10	-70.4351	-65.9584	4.4767	-1e+30 - 3	1.49223
*	12.5893	-63.3758	-62.2957	1.08013	-1e+30 - 3	0.360042
*	15.8489	-56.6927	-56.4767	0.21595	-1e+30 - 3	0.0719832
*	19.9526	-50.4566	-50.4649	-0.0082744	-3 - 3	0.00275813
*	25.1189	-44.7072	-44.7474	-0.0401674	-2 - 2	0.0200837
*	31.6228	-39.444	-39.474	-0.0299704	-1.5 - 1.5	0.0199803
*	39.8107	-34.6341	-34.6581	-0.0240063	-1.5 - 1.5	0.0160042
*	50.1187	-30.2318	-30.2474	-0.0156154	-1.5 - 1.5	0.0104102
*	63.0957	-26.1976	-26.2057	-0.00810953	-1.5 - 1.5	0.00540635
*	79.4328	-22.5066	-22.5093	-0.00273116	-1.5 - 1.5	0.00182077
*	100	-19.1452	-19.1441	0.00106826	-1 - 1	0.00106826
*	125.893	-16.1003	-16.0967	0.00361809	-1 - 1	0.00361809
*	158.489	-13.3518	-13.3465	0.00532167	-1 - 1	0.00532167
*	199.526	-10.8715	-10.8654	0.00608995	-1 - 1	0.00608995
*	251.189	-8.63114	-8.62458	0.00656555	-1 - 1	0.00656555
*	316.228	-6.61163	-6.60528	0.00635681	-1 - 1	0.00635681
*	398.107	-4.80889	-4.80303	0.00585279	-1 - 1	0.00585279
*	501.187	-3.23297	-3.22842	0.00455615	-1 - 1	0.00455615
*	630.957	-1.90067	-1.89793	0.00274117	-1 - 1	0.00274117
*	794.328	-0.823951	-0.823745	0.000206604	-1 - 1	0.000206604
*	1000	0	0	0	-1 - 1	0
*	1258.93	0.591255	0.584398	-0.00685653	-1 - 1	0.00685653
*	1584.89	0.980885	0.967747	-0.0131372	-1 - 1	0.0131372
*	1995.26	1.20024	1.17845	-0.0217923	-1 - 1	0.0217923

*	2511.89	1.27114	1.23647	-0.0346677	-1 - 1	0.0346677
*	3162.28	1.19934	1.14602	-0.0533148	-1 - 1	0.0533148
*	3981.07	0.970799	0.890808	-0.0799911	-1 - 1	0.0799911
*	5011.87	0.549976	0.434195	-0.115781	-1.5 - 1.5	0.0771875
*	6309.57	-0.119329	-0.278225	-0.158896	-2 - 1.5	0.0794482
*	7943.28	-1.10806	-1.31012	-0.202057	-3 - 1.5	0.0673524
*	10000	-2.48833	-2.71556	-0.227224	-4 - 2	0.0568059
*	12589.3	-4.31312	-4.51208	-0.198958	-6 - 3	0.0331597
*	15848.9	-6.59725	-6.66759	-0.0703446	-1e+30 - 3	7.03446e-32
*	19952.6	-9.31078	-9.08017	0.230616	-1e+30 - 3	0.0768721

Correction: 2.24222 db, amp. factor 1.29453 (th. cor=1.2589)

Maximum relative error in decibels: 1.49223

Table 2.12: Response of corrected digital A weighting filter at sampling period 10^{-5}s with limited (**11 bits**) signal representation (see explanation of **Table 2.9**). Error is acceptable for a type 1 instrument and hardly acceptable for a type 2 instrument.

Corrected digital A filter, Ts=1e-5 s, limited representation (11 bits), amplitude of sinusoidal signals equals amplitude range:

	f [Hz]	an. r. [db]	num. dig.	n.d.-a.	tol. type 1	(n.d.-a.)/+tol.
*	10	-70.4351	-69.2649	1.17024	-1e+30 - 3	0.39008
*	12.5893	-63.3758	-63.2056	0.170192	-1e+30 - 3	0.0567308
*	15.8489	-56.6927	-56.6406	0.0520146	-1e+30 - 3	0.0173382
*	19.9526	-50.4566	-50.5003	-0.0436708	-3 - 3	0.0145569
*	25.1189	-44.7072	-44.7526	-0.0453987	-2 - 2	0.0226994
*	31.6228	-39.444	-39.4767	-0.0326585	-1.5 - 1.5	0.0217724
*	39.8107	-34.6341	-34.6554	-0.0212747	-1.5 - 1.5	0.0141832
*	50.1187	-30.2318	-30.2432	-0.0114729	-1.5 - 1.5	0.00764858
*	63.0957	-26.1976	-26.2014	-0.00379237	-1.5 - 1.5	0.00252825
*	79.4328	-22.5066	-22.5048	0.00179175	-1.5 - 1.5	0.0011945
*	100	-19.1452	-19.1396	0.00552147	-1 - 1	0.00552147
*	125.893	-16.1003	-16.092	0.00829559	-1 - 1	0.00829559
*	158.489	-13.3518	-13.3418	0.00998063	-1 - 1	0.00998063
*	199.526	-10.8715	-10.8608	0.0107419	-1 - 1	0.0107419
*	251.189	-8.63114	-8.61998	0.0111636	-1 - 1	0.0111636
*	316.228	-6.61163	-6.60056	0.0110711	-1 - 1	0.0110711
*	398.107	-4.80889	-4.79842	0.0104627	-1 - 1	0.0104627
*	501.187	-3.23297	-3.22372	0.00925722	-1 - 1	0.00925722
*	630.957	-1.90067	-1.89336	0.00730831	-1 - 1	0.00730831
*	794.328	-0.823951	-0.819084	0.00486669	-1 - 1	0.00486669
*	1000	0	0	0	-1 - 1	0
*	1258.93	0.591255	0.588997	-0.00225808	-1 - 1	0.00225808
*	1584.89	0.980885	0.972343	-0.00854108	-1 - 1	0.00854108
*	1995.26	1.20024	1.18313	-0.0171118	-1 - 1	0.0171118
*	2511.89	1.27114	1.24114	-0.0299934	-1 - 1	0.0299934
*	3162.28	1.19934	1.15059	-0.0487478	-1 - 1	0.0487478
*	3981.07	0.970799	0.895437	-0.075362	-1 - 1	0.075362
*	5011.87	0.549976	0.43888	-0.111097	-1.5 - 1.5	0.0740644
*	6309.57	-0.119329	-0.273638	-0.154309	-2 - 1.5	0.0771545
*	7943.28	-1.10806	-1.3055	-0.19744	-3 - 1.5	0.0658135
*	10000	-2.48833	-2.7127	-0.224365	-4 - 2	0.0560912
*	12589.3	-4.31312	-4.50743	-0.194314	-6 - 3	0.0323856
*	15848.9	-6.59725	-6.66297	-0.065724	-1e+30 - 3	6.5724e-32
*	19952.6	-9.31078	-9.07554	0.235245	-1e+30 - 3	0.0784151

Correction: 2.24689 db, amp. factor 1.29522 (th. cor=1.2589)

Maximum relative error in decibels: 0.39008

Table 2.13: Response of corrected digital A weighting filter at sampling period 10^{-5}s with limited (**12 bits**) signal representation (see explanation of **Table 2.9**). Error because of discrete level representation is still present and vanishes at 13 bits. Error can however be considered acceptable for a type 1 and 2 instrument.

```
Corrected digital A filter, Ts=1e-5 s, limited representation (12 bits),
amplitude of sinusoidal signals equals amplitude range:


Corrected digital response, correction=2.24466 db:
    f [Hz]  an. r. [db]    num. dig.      n.d.-a.  tol. type 1 (n.d.-a.)/+tol.
*       10     -70.4351     -69.9916     0.443546 -1e+30 - 3       0.147849
*  12.5893     -63.3758     -63.3869    -0.011094 -1e+30 - 3     1.1094e-32
*  15.8489     -56.6927     -56.7437   -0.0510827 -1e+30 - 3    5.10827e-32
*  19.9526     -50.4566     -50.5219   -0.0652694    -3 - 3      0.0217565
*  25.1189     -44.7072     -44.7579   -0.0506681    -2 - 2      0.0253341
*  31.6228      -39.444     -39.4808   -0.0367662  -1.5 - 1.5    0.0245108
*  39.8107     -34.6341     -34.6581   -0.0240096  -1.5 - 1.5    0.0160064
*  50.1187     -30.2318     -30.2455   -0.0137465  -1.5 - 1.5   0.00916435
*  63.0957     -26.1976     -26.2037  -0.00612825  -1.5 - 1.5    0.0040855
*  79.4328     -22.5066     -22.5071 -0.000497661  -1.5 - 1.5   0.000331774
*      100     -19.1452     -19.1418   0.00337538    -1 - 1     0.00337538
*  125.893     -16.1003     -16.0943   0.00602782    -1 - 1     0.00602782
*  158.489     -13.3518     -13.3441   0.00771941    -1 - 1     0.00771941
*  199.526     -10.8715      -10.863   0.00850218    -1 - 1     0.00850218
*  251.189     -8.63114     -8.62223   0.00891181    -1 - 1     0.00891181
*  316.228     -6.61163     -6.60287   0.00876745    -1 - 1     0.00876745
*  398.107     -4.80889     -4.80067   0.00821785    -1 - 1     0.00821785
*  501.187     -3.23297     -3.22599   0.00698371    -1 - 1     0.00698371
*  630.957     -1.90067     -1.89562   0.00504857    -1 - 1     0.00504857
*  794.328    -0.823951    -0.821369   0.00258172    -1 - 1     0.00258172
*     1000            0            0            0    -1 - 1              0
*  1258.93     0.591255     0.586764  -0.00449079    -1 - 1     0.00449079
*  1584.89     0.980885     0.970085   -0.0107995    -1 - 1      0.0107995
*  1995.26      1.20024      1.18087   -0.0193698    -1 - 1      0.0193698
*  2511.89      1.27114      1.23885   -0.0322875    -1 - 1      0.0322875
*  3162.28      1.19934      1.14833   -0.0510007    -1 - 1      0.0510007
*  3981.07     0.970799     0.893177   -0.0776221    -1 - 1      0.0776221
*  5011.87     0.549976     0.436659    -0.113317  -1.5 - 1.5    0.0755449
*  6309.57    -0.119329    -0.275846    -0.156518    -2 - 1.5    0.0782588
*  7943.28     -1.10806     -1.30772    -0.199661    -3 - 1.5    0.0665537
*    10000     -2.48833     -2.71174     -0.22341    -4 - 2      0.0558526
*  12589.3     -4.31312     -4.50972    -0.196602    -6 - 3       0.032767
*  15848.9     -6.59725     -6.66525    -0.068001 -1e+30 - 3     6.8001e-32
*  19952.6     -9.31078     -9.07784     0.232948 -1e+30 - 3      0.0776495

Correction: 2.24466 db, amp. factor 1.29489 (th. cor=1.2589)
```

Maximum relative error in decibels: 0.147849

Table 2.14: Response of corrected digital A weighting filter at sampling period 10^{-5}s with very low (**4 bits where 8 positive and negative levels can be represented**) signal resolution (see explanation of **Table 2.9**). Filtere response is completely destroyed for frequencies below 100 Hz.

Corrected digital A filter, Ts=1e-5 s, limited representation (4 bits)>

	f [Hz]	an. r. [db]	num. dig.	n.d.-a.	tol. type 1	(n.d.-a.)/+tol.
*	10	-70.4351	-29.1868	41.2484	-1e+30 - 3	13.7495
*	12.5893	-63.3758	-28.4138	34.962	-1e+30 - 3	11.654
*	15.8489	-56.6927	-27.7283	28.9644	-1e+30 - 3	9.65479
*	19.9526	-50.4566	-27.14	23.3165	-3 - 3	7.77218
*	25.1189	-44.7072	-26.6404	18.0668	-2 - 2	9.03341
*	31.6228	-39.444	-26.1744	13.2696	-1.5 - 1.5	8.84642
*	39.8107	-34.6341	-25.6067	9.02736	-1.5 - 1.5	6.01824
*	50.1187	-30.2318	-24.6946	5.53718	-1.5 - 1.5	3.69145
*	63.0957	-26.1976	-23.1781	3.01949	-1.5 - 1.5	2.01299
*	79.4328	-22.5066	-21.0164	1.49024	-1.5 - 1.5	0.993492
*	100	-19.1452	-18.449	0.696154	-1 - 1	0.696154
*	125.893	-16.1003	-15.7837	0.316654	-1 - 1	0.316654
*	158.489	-13.3518	-13.2111	0.140693	-1 - 1	0.140693
*	199.526	-10.8715	-10.8121	0.0594233	-1 - 1	0.0594233
*	251.189	-8.63114	-8.611	0.0201433	-1 - 1	0.0201433
*	316.228	-6.61163	-6.61009	0.00153989	-1 - 1	0.00153989
*	398.107	-4.80889	-4.81647	-0.00758056	-1 - 1	0.00758056
*	501.187	-3.23297	-3.24556	-0.0125844	-1 - 1	0.0125844
*	630.957	-1.90067	-1.91812	-0.0174486	-1 - 1	0.0174486
*	794.328	-0.823951	-0.845145	-0.0211939	-1 - 1	0.0211939
*	1000	0	0	0	-1 - 1	0
*	1258.93	0.591255	0.562771	-0.0284839	-1 - 1	0.0284839
*	1584.89	0.980885	0.946065	-0.0348192	-1 - 1	0.0348192
*	1995.26	1.20024	1.15819	-0.0420459	-1 - 1	0.0420459
*	2511.89	1.27114	1.21649	-0.0546433	-1 - 1	0.0546433
*	3162.28	1.19934	1.12404	-0.0752919	-1 - 1	0.0752919
*	3981.07	0.970799	0.871585	-0.0992135	-1 - 1	0.0992135
*	5011.87	0.549976	0.415032	-0.134944	-1.5 - 1.5	0.0899628
*	6309.57	-0.119329	-0.298052	-0.178724	-2 - 1.5	0.0893618
*	7943.28	-1.10806	-1.32965	-0.221587	-3 - 1.5	0.0738624
*	10000	-2.48833	-2.62558	-0.137243	-4 - 2	0.0343108
*	12589.3	-4.31312	-4.52824	-0.215119	-6 - 3	0.0358532
*	15848.9	-6.59725	-6.68114	-0.0838912	-1e+30 - 3	8.38912e-32
*	19952.6	-9.31078	-9.08381	0.226972	-1e+30 - 3	0.0756573

Correction: 2.17788 db, amp. factor 1.28497 (th. cor=1.2589)

Maximum relative error in decibels: 13.7495

```
++++                                         X                                #
    +++++++                                   X                               #
            ++++++                            X                               #
                   ++++++                      X                              #
5                        ++++++                 X                             #
6                              +++++            X                             #
(7,-25.6067)                         ++++        X                            #
(8,-24.6946)                              ++++    X                           #
(9,-23.1781)                                  ++++ X                          #
(10,-21.0164)                                     +++*X                       #
(11,-18.449)                                          ++*X                    #
(12,-15.7837)                                             *X                  #
(13,-13.2111)                                                *X               #
(14,-10.8121)                                                   *X            #
(15,-8.611)                                                       *X          #
(16,-6.61009)                                                       *X        #
(17,-4.81647)                                                         *X      #
(18,-3.24556)                                                           *X    #
(19,-1.91812)                                                              *X #
(20,-0.845145)                                                               X#
(21,0)                                                                        X
(22,0.562771)                                                                 X
```

```
(23,0.946065)                                                                #X
(24,1.15819)                                                                 #X
(25,1.21649)                                                                 #X
(26,1.12404)                                                                 #X
(27,0.871585)                                                                #X
(28,0.415032)                                                                X
(29,-0.298052)                                                              X#
(30,-1.32965)                                                              X #
(31,-2.62558)                                                             X  #
(32,-4.52824)                                                           X*   #
(33,-6.68114)                                                        X*      #
(34,-9.08381)                                                     +X*        #
```

2.4.3.1 Smoothing of Sampled Signals

Limited level resolution causes problems because a continuously varying signal is represented by a step function. Even when the signal is gradually changing, a number of samples have the same discrete level, which then jumps to another discrete level, and after a number of samples of the same represented level it jumps to another discrete level again (Figure 2.5). This behaviour is emphasised at low frequencies where the number of consequent samples with the same level is greater because of slower variation by time.

One idea of how to correct the error in response of digital filters because of the discrete level representation is therefore smoothing of the sample before filtering.

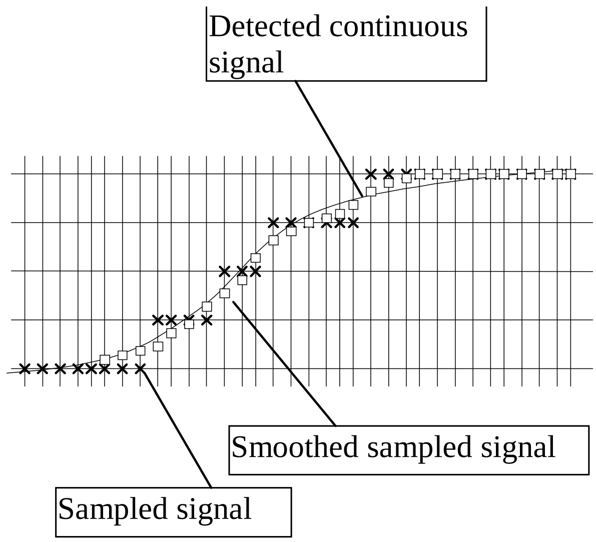

Figure 2.5: effects of sampling with AD conversion and smoothing.

Smoothing should result in intermediate levels of samples as shown in Figure 2.5. Computationally acceptable approaches include various ways of averaging, where each sample value is obtained as a combination of values of neighbouring samples. Smoothed samples must of course be represented by higher resolution than is used for AD conversion.

Two basic approaches were tested. At the first approach the level of each smoothed sample is a combination of levels of a certain number of previous non-smoothed samples. The sum of coefficient of the linear combination must be 1. At low frequencies this approach is not very efficient unless a high number of previous samples are involved.

Another approach each smoothed sample is a combination of a certain number of previous already smoothed samples. This approach is more efficient because the effect of smoothing of a given sample stretches back to all previous samples.

Response of frequency weighting filter is much better on smoothed signals than on non-smoothed. Various numbers of smoothing parameters have been optimised in order to give the best filter response on smoothed signals. Results are shown in Table 2.15 for the second approach, where smoothed signals are combinations of already smoothed signals.

Table 2.15: Maximum relative errors of the A weighting filter response on optimally smoothed samples with certain level resolution. Different number of smoothing parameters were used.

Bits/Par.	7	5	3	2	No smoothing
8	1.4	0.33	0.41	0.44	6.4
9	0.51	0.085	0.25	0.27	3.75
10	0.099	0.105	0.084	0.088	1.49
11	0.205	0.085	0.078	0.078	0.39
12	0.099	0.085	0.078	0.078	0.15

These results show that errors with respect to the required response are much smaller on smoothed signals than on non-smoothed. The problem is that smoothing itself produces a frequency dependent attenuation of the signal (higher frequencies are attenuated more than lower ones). Combined frequency response of the smoothing and weighting filter therefore differs from original response. The frequency response of the weighting filter should therefore be corrected in such a way that the frequency response of smoothing would be compensated.

A larger number of smoothing parameters and filter corrections were optimised in order to achieve smoothing and correct frequency response at the same time. Results are rather poor and lead to conclusion that smoothing is not worth to be applied, since a minimal gain in reducing the level discretisation error can not excuse the additional processing cost.

Table 2.16: Maximum relative errors of the optimally combined smoothing and A weighting. Different number of smoothing parameters were used. When 11 parameters are used, there are 1 amplitude factor, 5 smoothing parameters for the first approach, 3 smoothing parameters for the second approach, and 2 frequency corrections for the weighting filter. When 15 parameters are used, there are 1 amplitude factor, 5 smoothing parameters for the first approach, 5 smoothing parameters for the second approach, and 4 frequency corrections for the weighting filter.

Bits/Par.	11	15	No smoothing
8	4.77		6.4
9	2.53	2.316	3.75
10	1.016	0.743	1.49
11	0.304	0.182	0.39
12	0.125		0.15

2.5 Tables from Standards

Table 2.17: A, B and C weighted frequency response in decibels as specified in the standard [5]. *n* is a frequency measure, namely the frequency is obtained as $f(n) = 1KHz \cdot 10^{0.1n}$.

n	nom.f.	A wt.	B wt.	C weighting
-20	10	-70.4	-38.2	-14.3
-19	12.5	-63.4	-33.2	-11.2
-18	16	-56.7	-28.5	-8.5
-17	20	-50.5	-24.2	-6.2
-16	25	-44.7	-20.4	-4.4
-15	31.5	-39.4	-17.1	-3.0
-14	40	-34.6	-14.2	-2.0
-13	50	-30.2	-11.6	-1.3
-12	63	-26.2	-9.3	-0.8
-11	80	-22.5	-7.4	-0.5
-10	100	-19.1	-5.6	-0.3
-9	125	-16.1	-4.2	-0.2
-8	160	-13.4	-3.0	-0.1
-7	200	-10.9	-2.0	-0.0
-6	250	-8.6	-1.3	-0.0
-5	315	-6.6	-0.8	-0.0
-4	400	-4.8	-0.5	-0.0
-3	500	-3.2	-0.3	-0.0

-2	630	-1.9	-0.1	-0.0
-1	800	-0.8	-0	-0.0
0	1000	0	0	0
1	1250	0.6	-0	-0.0
2	1600	1.0	-0	-0.1
3	2000	1.2	-0.1	-0.2
4	2500	1.3	-0.2	-0.3
5	3150	1.2	-0.4	-0.5
6	4000	1.0	-0.7	-0.8
7	5000	0.5	-1.2	-1.3
8	6300	-0.1	-1.9	-2.0
9	8000	-1.1	-2.9	-3.0
10	10000	-2.5	-4.3	-4.4
11	12500	-4.2	-6.1	-6.2
12	16000	-6.6	-8.4	-8.5
13	20000	-9.3	-11.1	-11.2

Table 2.18: Admitted tolerances for sound level meters of type 0, 1 and 2.

tolerances

n	nom.f	0-	0+	1-	1+	2-	2+
-20	10	-1e30	2	-1e30	3	-1e30	5
-19	12.5	-1e30	2	-1e30	3	-1e30	5
-18	16	-1e30	2	-1e30	3	-1e30	5
-17	20	-2	2	-3	3	-3	3
-16	25	-1.5	1.5	-2	2	-3	3
-15	31.5	-1	1	-1.5	1.5	-3	3
-14	40	-1	1	-1.5	1.5	-2	2
-13	50	-1	1	-1.5	1.5	-2	2
-12	63	-1	1	-1.5	1.5	-2	2
-11	80	-1	1	-1.5	1.5	-2	2
-10	100	-0.7	0.7	-1	1	-1.5	1.5
-9	125	-0.7	0.7	-1	1	-1.5	1.5
-8	160	-0.7	0.7	-1	1	-1.5	1.5
-7	200	-0.7	0.7	-1	1	-1.5	1.5
-6	200	-0.7	0.7	-1	1	-1.5	1.5
-5	315	-0.7	0.7	-1	1	-1.5	1.5
-4	400	-0.7	0.7	-1	1	-1.5	1.5
-3	500	-0.7	0.7	-1	1	-1.5	1.5
-2	630	-0.7	0.7	-1	1	-1.5	1.5
-1	800	-0.7	0.7	-1	1	-1.5	1.5
0	1000	-0.7	0.7	-1	1	-1.5	1.5
1	1250	-0.7	0.7	-1	1	-1.5	1.5
2	1600	-0.7	0.7	-1	1	-2	2
3	2000	-0.7	0.7	-1	1	-2	2
4	2500	-0.7	0.7	-1	1	-2.5	2.5
5	3150	-0.7	0.7	-1	1	-2.5	2.5
6	4000	-0.7	0.7	-1	1	-3	3
7	5000	-1	1	-1.5	1.5	-3.5	3.5
8	6300	-1.5	1	-2	1.5	-4.5	4.5
9	8000	-2	1	-3	1.5	-5	5
10	10000	-3	2	-4	2	-1e30	5
11	12500	-3	2	-6	3	-1e30	5
12	16000	-3	2	-1e30	3	-1e30	5
13	20000	-3	2	-1e30	3	-1e30	5

2.6 Time Weighting

After the signal is frequency weighted, time weighting must be applied (Figure 2.6). Either S (slow), F (fast) of I (impulse) weighting regime is used.

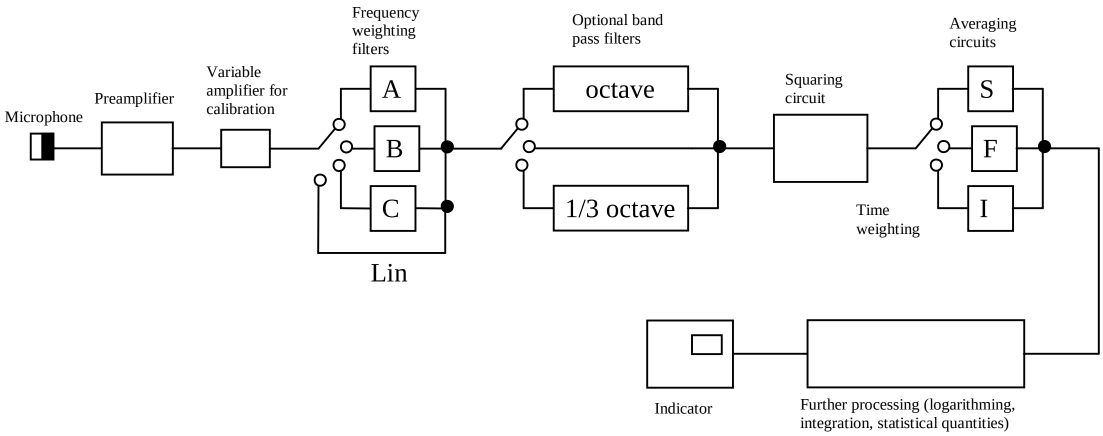

Figure 2.6: Scheme of a complete sound level meter.

Time weighting consists of a squaring circuit and an exponential averaging circuit (Figure 2.7). In the case of digital implementation, the squaring unit simply squares each sample. The **exponential averaging circuit** is implemented as a **low pass filter** (either analogue or digital, dependent on implementation).

The exponential averaging circuits differ in time constant for F, S and I weighting. This constant determines how quickly the indication (or output signal) falls when a constant signal is suddenly switched off. For a low pass filter, the time constant is

$$\underline{\tau = \frac{1}{RC} = \frac{1}{\omega_0} = \frac{1}{2\pi f_0}}, \tag{2.44}$$

therefore the characteristic frequency of the appropriate low pass filter must be

$$f_0 = \frac{1}{2\pi\tau}. \tag{2.45}$$

The decay of the indication when turning off a constant input signal is described by

$$O(t) = O(0)e^{-t/\tau} \tag{2.46}$$

or in the case of a digital signal

$$O_{N+i} = O_N \, e^{-iT/\tau}. \tag{2.47}$$

This gives a decay for factor $e^{-1s/\tau}$ per a second, which can be expressed in decibels by taking the base ten logarithm and then multiplying by a factor 10.

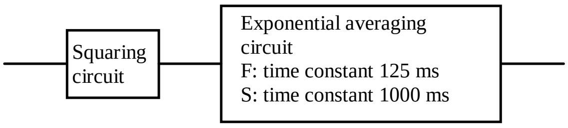

Figure 2.7: Scheme of F and S time weighting

The impulse time weighting includes, in addition to the squaring and averaging circuit, the **peak detector** (Figure 2.8). The peak detector has a function of storing the voltage fed to it for a sufficient time to allow it to be displayed by the indicator. The onset time of the peak detector must be small compared with the time constant of the averaging circuit (35 ms). Its Decay rate must be 2.9 dB/s, which gives the time constant $\tau_{peak} \approx 1.5\,s$.

In digital implementation, the peak detector is simply a unit whose *i*-th output is set either to its input or to the decayed previous output, whichever is greater in magnitude:

$$O_i = \begin{cases} I_i\,; & I_i \geq O_i \, e^{-T/\tau_{peak}} \\ O_i \, e^{-T/\tau_{peak}} & ;\, otherwise \end{cases} \tag{2.48}$$

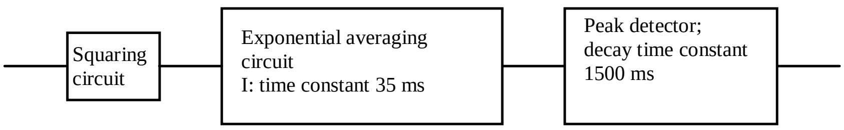

Figure 2.8: Scheme of I time weighting

2.7 Indication

Indication of sound levels must be in decibels with respect to the reference sound pressure or intensity (see Section 1.1). The sound pressure at the microphone is converted to voltage, which is then additionally amplified. Let us say that the ratio between the pressure on the microphone and the voltage that is frequency weighted is $K_m\,[V/Pa]$, which is the sensitivity of microphone with the amplifier. The output of the time weighting unit is converted to the indicated sound level in decibels as

$$L = 10\log_{10}\frac{O_i}{(K_m p_0)^2}\,, \tag{2.49}$$

where O_i is the current output (in volts) of the time weighting unit and p_0 is the reference sound pressure of $20\mu Pa$. Note that O_i contains squared and averaged signal (i.e. a kind of r.m.s. of the signal).

It is specified[5] that the range of the indicator must be at least 15 dB. Resolution of the digital display must be 0.1 dB.

2.8 Calculation of Integral and Statistical Quantities

Integral quantities can be evaluated by replacing the integral by a weighted sum. Weights can be uniform provided that the sampling interval of summation is short enough. The interval of $1\,ms$ is recommended, which means that every hundredth sample that contains the integrated quantity is added to the sum at the sampling rate of $10^5\ s^{-1}$. Integration of a quantity *A(t)* is therefore performed as

$$\int_{T_1}^{T_2} A(t)dt := 100T\sum_{i=1}^{(T_2-T_1)/(100T)} A(100\,i\,T)\,. \tag{2.50}$$

Statistical quantities L_x refer to the level of noise that is exceeded in a certain portion of the measuring time (e.g., L_{90} is the level of noise that was exceeded in 90% of the measuring time). In order to measure these quantities, we must divide the total possible range of measured levels into a large enough number of intervals and count for each of them the number of events during the measuring time when the level of noise falls into that interval. The total range will cover all measuring ranges of the instrument. Number of intervals will depend on the required accuracy and the total range. For example, if the total range is from 0 to 100 dB and the required accuracy is 0.5 dB, we must count the number of appearances of a specific level for 200 intervals.

Measuring statistical quantities requires allocation of a table that holds counters for all intervals of the level of noise that are monitored. Use of 32-bit integers will satisfy the requirements for the instrument. Updating counters does not need to be performed on every sample. We must update the counters on uniform time periods in which the time weighted level can not change too much. If the sampling rate is $10^5 s^{-1}$, we can update counters at every 100-th sample.

The updating procedure consists of taking the measured noise level on uniform periods of time (e.g. every 100-th sample), calculating the index of interval into which the level falls (total range divided by the number of intervals) and incrementing the appropriate counter.

When evaluating the quantity L_x, we must first form the cumulative table of counters from the table which count number of events for individual intervals. For example, if we have table of events *T* and the cumulative table *C*, both with *n* elements, then elements of the cumulative table are evaluated in the following way:

```
C[1]:= T[1];
for (i:=2 to n)
     C[i]:= C[i-1]+T[i] ;
```

If the statistical quantities will be calculated only at the end of the measurement, then the cumulative table *C* can be stored in the same space as the table of counters *T*, otherwise a special storage is required. When we have the cumulative table, the appropriate statistical quantity is evaluated by finding index of the last element of the cumulative table which contains less than a given percentage of the total number of counts (which equals to the contents of the last element of the cumulative table). This index is then converted to the corresponding noise level and evaluation of the statistical quantity is done.

For example, the index k for L_x would be calculated in the following way:

```
kx:=0;
for (i:=1 to n)
     if (C[i]<(1-x)*C[n])
            kx:=i;
     else
            exit the for loop
```

For L_{90} x in the above code will be 0.9, for example. The value of L_x is obtained by multiplying the calculated index *k* by the total range, which corresponds to the last interval and therefore to the last element of tables *T* and *L* (e.g. 100 dB).

In practice, of course, more than one indices that determine the corresponding statistical quantities, will be calculated at once. For example, if wee need the quantities L_{10} and L_{90}, they will be calculated in the following way:

```
k90:=0;
k10:=0;
for (i:=1 to n)
     if (C[i]<(1-0.9)*C[n])
           k90:=i;
     else
           exit the for loop
for (i:=k90 to n)
     if (C[i]<(1-0.1)*C[n])
           k10:=i;
     else
           exit the for loop
L90:=k90*total_range;
L10:=k10*total_range;
```

3 Octave-band and One-third-octave-band Filters

3.1 Introduction

The standard[9] specifies properties of the band pass filters, which can be attached to the sound level meter. Lower and upper bounds on frequency response of these filters are specified and are listed in Table 3.1 and shown graphically in Figure 3.1.

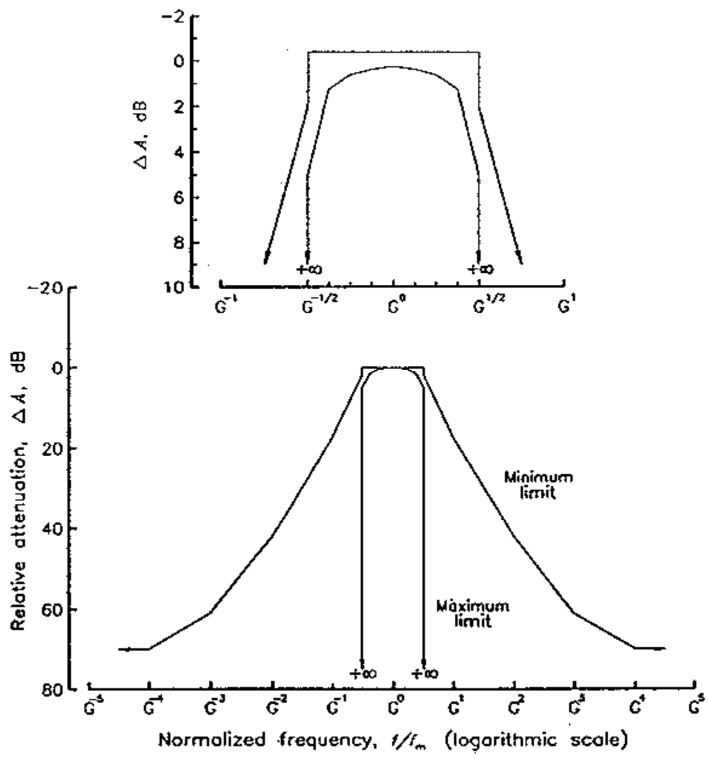

Figure 3.1: Graphical illustration of admitted bounds on response of octave band filters.

Table 3.1: Prescribed lower and upper bounds on octave-band filter response (permeability in decibels) for sound level meters of type 0, 1 and 2. Index *i* is a measure of frequency, i.e. $f_i = f_{cent} 2^i$, where f_{cent} is the central frequency of the octave filter. Limits are symmetric, which means that minimum and maximum responses for negative indices are the same as for positive ones.

index	0-	0+	1-	1+	2-	2+
0	-0.15	0.15	-0.3	0.3	-0.5	0.5
0.125	-0.2	0.15	-0.4	0.3	-0.6	0.5
0.25	-0.4	0.15	-0.6	0.3	-0.8	0.5
0.375	-1.1	0.15	-1.3	0.3	-1.6	0.5
0.5	-4.5	0.15	-5	0.3	-5.5	0.5
0.5	-4.5	-2.3	-5	-2	-5.5	-1.6
1	-1e30	-18.0	-1e30	-17.5	-1e30	-16.5
2	-1e30	-42.5	-1e30	-42	-1e30	-41
3	-1e30	-62	-1e30	-61	-1e30	-55
4	-1e30	-75	-1e30	-70	-1e30	-60

For the purpose of construction of octave and fractional octave filters, some measure for error in response must be defined. Filter response does not satisfy the prescribed one if it does not lie between the lower and upper bound as specified by the standard. The error measure will therefore be defined in the following way:

$$er(f) = \max\left(R(f) - R_{high}(f),\, R_{low}(f) - R(f)\right) \tag{3.51}$$

Of interest is also the maximum of this error measure over all frequencies,

$$maxer = \max_f er(f) \tag{3.52}$$

The filter response meets the requirements of the standard if $er(f) \le 0 \forall f$ or, equivalently, if $maxer \le 0$. The lowest the maximum error measure *maxer* is, the better the filter is.

It can be seen that very sharp transitions in response are required, which makes these filters difficult to implement. An analogue implementation of an octave band filter composed of high pass and low pass filters was searched for. It turned that it is practically impossible to construct such filter because of non-sharp transition in response of analogue low and high pass filters. For example, a filter with 13 (!) analogue low pass and high pass filters was tested. By changing parameters the smallest *maxer* achieved was 0.51 dB for type 1, which means that the filter was quite far from satisfying standard requirements. Data for this filter is listed in Table 3.2.

Table 3.2: Parameters for the best constructed filter composed of only analogue high and low pass filters. Each row in the table corresponds to a pair (or two pairs) of high and low pass filters. The characteristic frequency of the *i*-th high pass filter (or two filters when the right-most number is 2) is $f_h^{(i)} = f_{cent} 2^{balance - s_i}$ and the characteristic frequency of the corresponding low pass filter in the pair is $f_l^{(i)} = f_{cent} 2^{balance + s_i}$. The filter output must be amplified by *correction*.

i	s_i	Number of filters
1	0.0264894	1
2	-0.266864	1
3	-0.336659	1
4	0.662349	1
5	0.331586	1
6	1.11145	1
7	-0.366819	1
8	1.1968	2

9	1.40986	2
10	1.52029	2
Balance	0.00110773	
Correction	47.41 dB	
Maxer	0.510971 dB	

3.1.1 Some Basic Definitions

The standard [9] permits the base-ten and base-two options for determining an octave frequency ratio. With base-ten option the octave ratio is defined as

$$G_{10} = 10^{3/10} \approx 1.9953 \tag{3.53}$$

and with base-two option the octave ratio is defined as

$$G_2 = 2\,. \tag{3.54}$$

Throought this document the base-two option is adopted and notation $G = G_2$ is therefore used.

The adjacent filters in a set are characterised by their mid-band frequencies f_{cent}. One of these frequencies is the reference frequency of 1 kHz, denoted as

$$f_r = 1kHz\,. \tag{3.55}$$

Other mid-band frequencies can be obtained from

$$f_{cent}^{(x)} = G^{x/b} f_r\,, \tag{3.56}$$

where b designates the fraction of the octave band, i.e. $b = 1$ for octave-band filters and $b = 3$ for one-third-octave-band filters.

Filters are also characterised by bandedge frequencies (designated f_1 and f_2), i.e. the lower and upper edges of the passband defined in such a way that any two adjacent filters in a set share one bandedge frequency and that the mid-band frequency of the filter is the geometric mean of the lower and upper bandedge frequency. The lower and upper bandedge frequencies are determined from

$$f_1 = G^{-1/(2b)} f_{cent} \\ f_1 = G^{1/(2b)} f_{cent} \quad . \tag{3.57}$$

The normalised frequency will sometimes be used to state frequencies, and is defined as

$$\Omega = \frac{f}{f_{cent}}\,, \tag{3.58}$$

and sometimes the logarithmic frequency index will be used for this purpose, defined as

$$i = \log_2 \Omega = \log_2\left(f / f_{cent}\right). \tag{3.59}$$

3.1.2 Prescribed Limits on Permeability for Octave-band and One-third-octave-band filters

Table 3.1 shows the breakpoints in prescribed upper and lower limits on octave-band filter permeability. Between the breakpoints specified in the table, the limit change linearly with the logarithmic frequency. That is, the upper or lower limit A_x at the normalised frequency Ω_x is determined from relation

$$A_x = A_a + \left(A_b - A_a\right) \frac{\log\left(\Omega_x / \Omega_a\right)}{\log\left(\Omega_b / \Omega_a\right)}\,, \tag{3.60}$$

where A_a and A_b are the prescribed limits in two adjacent breakpoints from the table, and Ω_x, etc., are the corresponding normalised frequencies (see also Figure 3.1).

The breakpoint frequencies in prescribed limits for one-third-octave-band filters can be determined from the corresponding breakpoint frequencies for octave-band filters by using the relation

$$\Omega_h = 1 + \frac{G^{1/(2\cdot 3)} - 1}{G^{1/2} - 1}\left(\Omega - 1\right); \quad \Omega \geq 1\,, \tag{3.61}$$

where Ω is a normalised frequency of a breakpoint frequency for an octave-band filter and Ω_h is the normalised frequency of the corresponding breakpoint for a one-third-octave-band filter. Note that the corresponding breakpoint frequencies are not obdained simply by multiplication by a factor of $1/3$ in the logarithmic scale. However, the breakpoints located at the midband frequency and at both bandedge frequencies follow this rule.

Limit values in breakpoit frequencies for one-third-octave-band filters match the limit values for octave-band filters at the corresponding breakpoint frequencies, i.e. values listed in Table 3.1. For limits between the breakpoint frequencies the same relation as for octave-band filters applies (3.0, that is, the limit values between the breakpoints are linearly interpolated in the logarithmic frequency scale.

3.2 Resonance Filters

One idea is to use sharp resonance filters for construction of octave and fractional octave filters. The analogue resonance filter satisfies the following differential equation:

$$\frac{d^2 O(t)}{d\,t^2} + 2\beta \frac{d\,O(t)}{d\,t} + \omega_0^{\;2}\, O(t) = \omega_0^{\;2}\, I(t), \tag{3.62}$$

where *I(t)* is the input signal and *O(t)* is the output signal. Such a filter gives the following response:

$$\frac{|O_0|}{|I_0|} = \sqrt{\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(\frac{2\beta}{\omega_0}\right)^2\left(\frac{\omega}{\omega_0}\right)^2}\;. \tag{3.63}$$

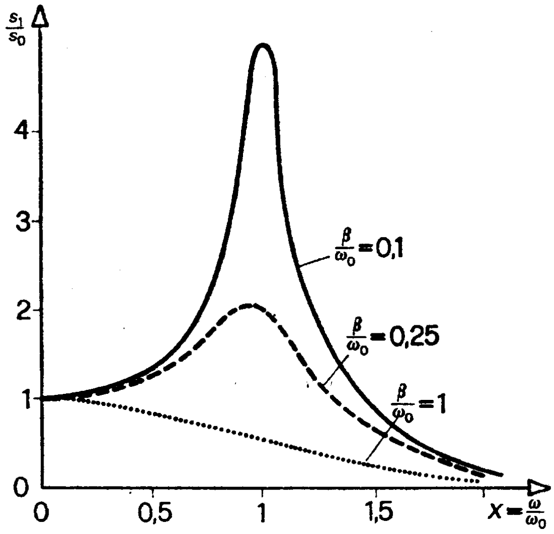

Figure 3.2: Shape of the resonance filter response for different values of β. Note that the frequency scale is linear, not logarithmic. The fact that the lower frequency response tends to 0 dB (i.e. the response is not symmetric about ω_0 in logarithmic scale) causes certain difficulties when using this filter in a band pass filter set.

3.2.1 Analogue Implementation of an Octave-band Filter By Using High Pass And Resonance Filters

An analogue filter was composed of 4 resonance and 4 high pass filters. The least maximum error measure achieved by such a filter was *maxer*=-0.288337 dB for type 1 bounds. This is a very good achievement if we take into account the fact that the best possible *maxer* for type 1 is –0.3 dB since the smallest gap in admitted response is 0.6 dB (see Table 3.1). The data for a filter is listed in Table 3.3.

The parameters listed in Table 3.3 have the following interpretation for resonance filters:

$$f_0^{(i)} = 2\pi\,\omega_0^{(i)} = f_{cent}\,2^{r_i}$$
$$\beta^{(i)} = 0.5\,h^{(i)}\omega_0^{(i)} \tag{3.64}$$

and the following interpretation for low pass filters:

$$f_0^{(i)} = f_{cent} 2^{k_i} \,. \tag{3.65}$$

Table 3.3: Data for analogue octave pass filter composed of 4 high pass and 4 resonance filters.

Resonance filters		
i	r_i	h_i
1	-0.380133	0.254101
2	0.0686898	0.419717
3	0.411579	0.222335
4	0.735172	0.221452
High pass filters		
i	k_i	
5	-0.5	
6	-0.5	
7	-1	
8	-1	

3.2.2 Digital Resonance Filters

A digital version of a resonance filter can be obtained by substituting the central difference derivative approximations

$$\begin{aligned} \left[\frac{\partial O}{\partial t}\right]_{i-1} &= \frac{O_i - O_{i-2}}{2T} \\ \left[\frac{\partial^2 O}{\partial t^2}\right]_{i-1} &= \frac{O_i - 2O_{i-1} + O_{i-2}}{T^2} \end{aligned} \tag{3.66}$$

in (3.13), which gives

$$\frac{O_i - 2O_{i-1} + O_{i-2}}{T^2} + 2\beta \frac{O_i - O_{i-2}}{2T} + \omega_0^{\,2} O_{i-1} = \omega_0^{\,2} I_{i-1} \tag{3.67}$$

Collecting terms with the same signals yields

$$\underbrace{\left(\frac{1}{T^2} + \frac{\beta}{T}\right)}_{\frac{1+\beta T}{T^2}} O_i = \frac{1}{T^2} I_{i-1} + \underbrace{\left(\frac{2}{T^2} - \omega_0^{\,2}\right)}_{\frac{2-\omega_0^{\,2}T^2}{T^2}} O_{i-1} + \underbrace{\left(\frac{\beta}{T} - \frac{1}{T^2}\right)}_{\frac{\beta T-1}{T^2}} O_{i-2},$$

and after division by the factor at O_i we have

$$\underline{\underline{O_i = \underbrace{\frac{\omega_0^{\,2} T^2}{1+\beta T}}_{a_1} I_{i-1} + \underbrace{\frac{2-\omega_0^{\,2} T^2}{1+\beta T}}_{b_1} O_{i-1} + \underbrace{\frac{\beta T - 1}{1+\beta T}}_{b_2} O_{i-2}}} \tag{3.68}$$

The above filter is determined by two parameters, the damping coefficient β and the central angular frequency ω_0. Instead of these two parameters we will usually state parameters

$$\underline{h_r = \frac{2\beta}{\omega_0}} \tag{3.69}$$

(the relative half-width of the resonance curve) and

$$\underline{f_0 = \frac{\omega_0}{2\pi}} \tag{3.70}$$

(the resonance frequency).

An alternative approach is to use the backward difference scheme for approximating derivatives. In this scheme formulas (3.11) approximate derivatives at the time iT instead of $(i-1)T$, therefore we have

$$\frac{O_i - 2O_{i-1} + O_{i-2}}{T^2} + 2\beta \frac{O_i - O_{i-2}}{2T} + \omega_0^{\,2} O_i = \omega_0^{\,2} I_i\,. \tag{3.71}$$

Collecting terms with the same signals gives

$$\underbrace{\left(\frac{1}{T^2}+\frac{\beta}{T}+\omega_0{}^2\right)}_{\frac{1+\beta T+\omega_0{}^2T^2}{T^2}}O_i=\omega_0{}^2 I_i+\frac{2}{T^2}O_{i-1}+\underbrace{\left(\frac{\beta}{T}-\frac{1}{T^2}\right)}_{\frac{\beta T-1}{T^2}}O_{i-2}\,,$$

and after division by the coefficient at O_i we finally have

$$O_i=\underbrace{\frac{\omega_0{}^2T^2}{1+\beta T+\omega_0{}^2T^2}}_{a_0}I_i+\underbrace{\frac{2}{1+\beta T+\omega_0{}^2T^2}}_{b_1}O_{i-1}+\underbrace{\frac{\beta T-1}{1+\beta T+\omega_0{}^2T^2}}_{b_2}O_{i-2}\,. \quad (3.72)$$

3.3 "Mirror Resonance" Filters

A drawback of the resonance filters is that their response is not symmetric (in logarithmic scale) about the central frequency. The response tends to 1 (0 dB) when the frequency approaches zero and decreases towards zero ($-\infty dB$) when the frequency increases above all limits. One possible solution to this problem is to use high pass filters in combination with resonance filters in order to attenuate lower frequencies.

Another solution, which turned more elegant, is to construct a filter whose response is approximately mirror symmetric in the logarithmic scale with respect to the response of the resonance filter about the central frequency (let us call it the "mirror resonance filter"), and then to combine both kinds of filters.

Mirror symmetry of filters with responses R and R_m about ω_0 means that

$$R(f)=R_m(f_m), \quad (3.73)$$

where

$$\frac{f}{f_0}=\frac{f_0}{f_m} \quad (3.74)$$

and therefore

$$f_m = \frac{f_0^{\ 2}}{f} \tag{3.75}$$

can be called the mirror frequency of f with respect to f_0.

$$\frac{d^2\, O(t)}{d\, t^2} + 2\beta\, \frac{d\, O(t)}{d\, t} + \omega_0^{\ 2}\, O(t) = \frac{d^2\, I(t)}{d\, t^2}\,, \tag{3.76}$$

where $I(t)$ is the input signal and $O(t)$ is the output signal. Such a filter gives the following response:

$$\frac{|O_0|}{|I_0|} = \sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(\frac{2\beta}{\omega_0}\right)^2 \left(\frac{\omega}{\omega_0}\right)^2}\ . \tag{3.77}$$

3.3.1 Digital Mirror Resonance Filters

A digital version of a resonance filter can be obtained by substituting the central difference derivative approximations

$$\begin{aligned} \left[\frac{\partial O}{\partial t}\right]_{i-1} &= \frac{O_i - O_{i-2}}{2T} \\ \left[\frac{\partial^2 O}{\partial t^2}\right]_{i-1} &= \frac{O_i - 2O_{i-1} + O_{i-2}}{T^2} \end{aligned} \tag{3.78}$$

in (3.0, which gives

$$\frac{O_i - 2O_{i-1} + O_{i-2}}{T^2} + 2\beta\, \frac{O_i - O_{i-2}}{2T} + \omega_0^{\ 2} O_{i-1} = \frac{I_i - 2I_{i-1} + I_{i-2}}{T^2} \tag{3.79}$$

Collecting terms with the same signals yields

$$\underbrace{\left(\frac{1}{T^2}+\frac{\beta}{T}\right)}_{\frac{1+\beta T}{T^2}}O_i=\frac{1}{T^2}I_i-\frac{2}{T^2}I_{i-1}+\frac{1}{T^2}I_{i-2}+\underbrace{\left(\frac{2}{T^2}-\omega_0{}^2\right)}_{\frac{2-\omega_0{}^2T^2}{T^2}}O_{i-1}+\underbrace{\left(\frac{\beta}{T}-\frac{1}{T^2}\right)}_{\frac{\beta T-1}{T^2}}O_{i-2},$$

and after division by the factor at O_i we have

$$\underline{\underline{O_i=\underbrace{\frac{1}{1+\beta T}}_{a_0}I_i-\underbrace{\frac{2}{1+\beta T}}_{a_1}I_{i-1}+\underbrace{\frac{1}{1+\beta T}}_{a_2}I_{i-2}+\underbrace{\frac{2-\omega_0{}^2T^2}{1+\beta T}}_{b_1}O_{i-1}+\underbrace{\frac{\beta T-1}{1+\beta T}}_{b_2}O_{i-2}}} \quad (3.80)$$

The above filter is determined by two parameters, the damping coefficient β and the central angular frequency ω_0. Instead of these two parameters we will usually state parameters

$$h_r=\frac{2\beta}{\omega_0} \quad (3.81)$$

(the relative half-width of the resonance curve) and

$$f_0=\frac{\omega_0}{2\pi} \quad (3.82)$$

(the resonance frequency).

An alternative approach is to use the backward difference scheme for approximating derivatives. In this scheme formulas (3.0 approximate derivatives at the time iT instead of $(i-1)T$, therefore we have

$$\frac{O_i-2O_{i-1}+O_{i-2}}{T^2}+2\beta\frac{O_i-O_{i-2}}{2T}+\omega_0{}^2O_i=\frac{I_i-2I_{i-1}+I_{i-2}}{T^2}\,. \quad (3.83)$$

Collecting terms with the same signals gives

$$\underbrace{\left(\frac{1}{T^2}+\frac{\beta}{T}+\omega_0{}^2\right)}_{\frac{1+\beta T+\omega_0{}^2T^2}{T^2}}O_i=\frac{1}{T^2}I_i-\frac{2}{T^2}I_{i-1}+\frac{1}{T^2}I_{i-2}+\frac{2}{T^2}O_{i-1}+\underbrace{\left(\frac{\beta}{T}-\frac{1}{T^2}\right)}_{\frac{\beta T-1}{T^2}}O_{i-2},$$

and after division by the coefficient at O_i we finally have

$$\begin{aligned} O_i = & \underbrace{\frac{1}{1+\beta T+\omega_0{}^2T^2}}_{a_0} I_i - \underbrace{\frac{2}{1+\beta T+\omega_0{}^2T^2}}_{a_1} I_{i-1} + \underbrace{\frac{1}{1+\beta T+\omega_0{}^2T^2}}_{a_2} I_{i-2} + \\ & + \underbrace{\frac{2}{1+\beta T+\omega_0{}^2T^2}}_{b_1} O_{i-1} + \underbrace{\frac{\beta T - 1}{1+\beta T+\omega_0{}^2T^2}}_{b_2} \end{aligned} \quad . \qquad (3.84)$$

3.4 Realisation of Digital Octave-band Filters

The following implementation of octave-band filters is proposed:

The sampled signal is subsequently filtered by two mirror resonance filters defined by (3.0, then by two resonance filters defined by (3.0, and finally multiplied by the corrective factor.

The calculated data for all octave-band filters is listed in Table 3.4. The data is valid for sampling rate of $10^5 s^{-1}$. For each filter the central frequency f_{cent} is specified first, designated *"fcent"*. Two pairs of parameters for mirror resonance filters and two pairs for resonance filters follow, and finally the corrective factor is stated, designated *"amp. factor"*.

For mirror resonance and resonance filters parameters designated *r1, r2, r5* and *r6* determine the characteristic frequencies (f_0 in (3.0 or (3.0) of the corresponding filters. Actual characteristic frequencies are obtained by multiplying the central frequency of the octave-band filter by 2 raised to the appropriate parameter, namely

$$f_0^{(i)} = f_{cent} \cdot 2^{r_i}, \qquad (3.85)$$

r_i being the parameter designated *r1, r2,* etc. Parameters designated *h1, h2, h5* and *h6* are correspond to the parameter h_r of each individual resonance or mirror resonance filter, as defined by (3.0 or (3.0. Coefficients for each individual resonance or mirror resonance filter are calculated from r_i and h_i, which are listed in Table 3.4, by using (3.0 and (3.0, and according to (3.0 for mirror resonance filters or according to (3.0 for resonance filters. Note again that two pairs of parameters for mirror resonance filters specified by (3.0 are listed first and then two pairs of parameters for resonance filters specified by (3.0 follow.

For each filter the error measure *maxerr* as is defined in (3.0 is stated in Table 3.4, designated *"Maximum relative error in decibels"*. Permeability at f_{cent} is also stated in the table and is designated *"Shift"*.

Table 3.4: parameters for octave-band filters.

```
fcent: 31.25

r1: -0.431245
h1: 0.222156

r2: 0.00080865
h2: 0.703445

r6: -0.0157486
h6: 0.663526

r7: 0.432015
h7: 0.204605

TOTAL correction: -19.3618 db, amp. factor 0.107624

Shift: -0.298956 db, amp. factor 0.966167

Maximum relative error in decibels: -0.298956
```

```
fcent: 62.5

r1: -0.511459
h1: 0.287905

r2: -0.198011
h2: 0.551898

r6: 0.369189
h6: 0.57036

r7: 0.523388
```

```
h7: 0.422391

TOTAL correction: -19.6575 db, amp. factor 0.104022

Shift: -0.301154 db, amp. factor 0.965923

Maximum relative error in decibels: -0.298846
```

```
fcent: 125

r1: -0.502608
h1: 0.280822

r2: -0.190728
h2: 0.553489

r6: 0.372174
h6: 0.565985

r7: 0.524362
h7: 0.422164

TOTAL correction: -19.7841 db, amp. factor 0.102517

Shift: -0.299912 db, amp. factor 0.966061

Maximum relative error in decibels: -0.299912
```

```
fcent: 250

r1: -0.490851
h1: 0.284555

r2: -0.195608
h2: 0.54981

r6: 0.386994
h6: 0.575294

r7: 0.501234
h7: 0.402489
```

```
TOTAL correction: -20.0384 db, amp. factor 0.0995589

Shift: -0.300353 db, amp. factor 0.966012

Maximum relative error in decibels: -0.299352
```

```
fcent: 500

r1: -0.500481
h1: 0.291873

r2: -0.21704
h2: 0.579537

r6: 0.291952
h6: 0.581422

r7: 0.534211
h7: 0.348228

TOTAL correction: -19.8636 db, amp. factor 0.101583

Shift: -0.299814 db, amp. factor 0.966072

Maximum relative error in decibels: -0.299814
```

```
fcent: 1000

r1: -0.480558
h1: 0.242194

r2: -0.172712
h2: 0.618881

r6: 0.193159
h6: 0.67413

r7: 0.448454
h7: 0.263902

TOTAL correction: -20.2872 db, amp. factor 0.0967471

Shift: -0.300186 db, amp. factor 0.96603

Maximum relative error in decibels: -0.299814
```

```
fcent: 2000

r1: -0.480843
h1: 0.235496

r2: -0.188683
h2: 0.616147

r6: 0.211596
h6: 0.673557

r7: 0.438767
h7: 0.269866


TOTAL correction: -20.4314 db, amp. factor 0.0951542

Shift: -0.299831 db, amp. factor 0.96607

Maximum relative error in decibels: -0.299831
```

```
fcent: 4000

r1: -0.482021
h1: 0.248642

r2: -0.178569
h2: 0.612204

r6: 0.195161
h6: 0.665947

r7: 0.453069
h7: 0.269671


TOTAL correction: -20.4634 db, amp. factor 0.0948047

Shift: -0.300183 db, amp. factor 0.966031

Maximum relative error in decibels: -0.299817
```

```
fcent: 8000

r1: -0.48875
h1: 0.274762

r2: -0.215401
h2: 0.697978

r6: 0.126905
h6: 0.669415

r7: 0.446389
h7: 0.275915


TOTAL correction: -19.704 db, amp. factor 0.103467

Shift: -0.299848 db, amp. factor 0.966068

Maximum relative error in decibels: -0.299848
```

```
fcent: 16000

r1: -0.510816
h1: 0.248242

r2: -0.248887
h2: 0.674746

r6: 0.108968
h6: 0.733439

r7: 0.359816
h7: 0.409083


TOTAL correction: -21.1552 db, amp. factor 0.0875464

Shift: -0.300118 db, amp. factor 0.966038

Maximum relative error in decibels: -0.299882
```

Permeability of the octave-band filter whose mid-band frequency is $f_{cent} = 1kHz$ is shown in Table 3.5.

Table 3.5: Response of the octave-band filter with $f_{cent} = 1000\,Hz$; filter data is listed in **Table 3.4**.

```
Corrected digital response, correction=-19.987 db:
   norm. f        f [Hz]    num. dig.  low. lim. -   up. lim.    errmin    errmax  error
*       -4          62.5     -108.614     -1e+30 - -70            -1e+30     -38.6  -38.614
   -3.9375       65.2671     -107.096     -1e+30 - -69.4375       -1e+30     -37.7  -37.659
*   -3.875       68.1567     -105.577     -1e+30 - -68.875        -1e+30     -36.7  -36.702
   -3.8125       71.1743     -104.058     -1e+30 - -68.3125       -1e+30     -35.7  -35.745
*    -3.75       74.3254     -102.536     -1e+30 - -67.75         -1e+30     -34.8  -34.786
   -3.6875       77.6161     -101.014     -1e+30 - -67.1875       -1e+30     -33.8  -33.826
*   -3.625       81.0525     -99.4893     -1e+30 - -66.625        -1e+30     -32.9  -32.864
   -3.5625        84.641     -97.9631     -1e+30 - -66.0625       -1e+30     -31.9  -31.901
*     -3.5       88.3883     -96.4351     -1e+30 - -65.5          -1e+30     -30.9  -30.935
   -3.4375       92.3016     -94.9051     -1e+30 - -64.9375       -1e+30       -30  -29.968
*   -3.375       96.3882     -93.3727     -1e+30 - -64.375        -1e+30       -29  -28.998
   -3.3125       100.656     -91.8378     -1e+30 - -63.8125       -1e+30       -28  -28.025
*    -3.25       105.112     -90.3003     -1e+30 - -63.25         -1e+30     -27.1  -27.05
   -3.1875       109.766     -88.7597     -1e+30 - -62.6875       -1e+30     -26.1  -26.072
*   -3.125       114.626      -87.216     -1e+30 - -62.125        -1e+30     -25.1  -25.091
   -3.0625         119.7     -85.6686     -1e+30 - -61.5625       -1e+30     -24.1  -24.106
*       -3           125     -84.1174     -1e+30 - -61            -1e+30     -23.1  -23.117
   -2.9375       130.534     -82.5619     -1e+30 - -59.8125       -1e+30     -22.7  -22.749
*   -2.875       136.313     -81.0017     -1e+30 - -58.625        -1e+30     -22.4  -22.377
   -2.8125       142.349     -79.4367     -1e+30 - -57.4375       -1e+30       -22  -21.999
*    -2.75       148.651      -77.866     -1e+30 - -56.25         -1e+30     -21.6  -21.616
   -2.6875       155.232     -76.2891     -1e+30 - -55.0625       -1e+30     -21.2  -21.227
*   -2.625       162.105     -74.7056     -1e+30 - -53.875        -1e+30     -20.8  -20.831
   -2.5625       169.282     -73.1147     -1e+30 - -52.6875       -1e+30     -20.4  -20.427
*     -2.5       176.777      -71.516     -1e+30 - -51.5          -1e+30       -20  -20.016
   -2.4375       184.603     -69.9084     -1e+30 - -50.3125       -1e+30     -19.6  -19.596
*   -2.375       192.776     -68.2914     -1e+30 - -49.125        -1e+30     -19.2  -19.166
   -2.3125       201.311     -66.6636     -1e+30 - -47.9375       -1e+30     -18.7  -18.726
*    -2.25       210.224     -65.0241     -1e+30 - -46.75         -1e+30     -18.3  -18.274
   -2.1875       219.532     -63.3719     -1e+30 - -45.5625       -1e+30     -17.8  -17.809
*   -2.125       229.251     -61.7059     -1e+30 - -44.375        -1e+30     -17.3  -17.331
   -2.0625       239.401     -60.0241     -1e+30 - -43.1875       -1e+30     -16.8  -16.837
*       -2           250     -58.3254     -1e+30 - -42            -1e+30     -16.3  -16.325
   -1.9375       261.068     -56.6078     -1e+30 - -40.4688       -1e+30     -16.1  -16.139
*   -1.875       272.627     -54.8694     -1e+30 - -38.9375       -1e+30     -15.9  -15.932
   -1.8125       284.697     -53.1081     -1e+30 - -37.4062       -1e+30     -15.7  -15.702
*    -1.75       297.302     -51.3215     -1e+30 - -35.875        -1e+30     -15.4  -15.446
   -1.6875       310.464     -49.5062     -1e+30 - -34.3438       -1e+30     -15.2  -15.162
*   -1.625        324.21     -47.6599     -1e+30 - -32.8125       -1e+30     -14.8  -14.847
   -1.5625       338.564     -45.7781     -1e+30 - -31.2812       -1e+30     -14.5  -14.497
*     -1.5       353.553     -43.8576     -1e+30 - -29.75         -1e+30     -14.1  -14.108
   -1.4375       369.207     -41.8933     -1e+30 - -28.2188       -1e+30     -13.7  -13.675
*   -1.375       385.553     -39.8802     -1e+30 - -26.6875       -1e+30     -13.2  -13.193
   -1.3125       402.623     -37.8118     -1e+30 - -25.1562       -1e+30     -12.7  -12.656
*    -1.25       420.448     -35.6812     -1e+30 - -23.625        -1e+30     -12.1  -12.056
   -1.1875       439.063     -33.4799     -1e+30 - -22.0938       -1e+30     -11.4  -11.386
*   -1.125       458.502     -31.1987     -1e+30 - -20.5625       -1e+30     -10.6  -10.636
   -1.0625       478.802     -28.8256     -1e+30 - -19.0312       -1e+30     -9.79  -9.7943
*       -1           500     -26.3476     -1e+30 - -17.5          -1e+30     -8.85  -8.8476
   -0.9375       522.137     -23.7494  -8.75e+29 - -15.5625    -8.75e+29     -8.19  -8.1869
*   -0.875       545.254     -21.0133   -7.5e+29 - -13.625      -7.5e+29     -7.39  -7.3883
   -0.8125       569.394     -18.1217  -6.25e+29 - -11.6875    -6.25e+29     -6.43  -6.4342
*    -0.75       594.604     -15.0614     -5e+29 - -9.75          -5e+29     -5.31  -5.3114
   -0.6875       620.929     -11.8391  -3.75e+29 - -7.8125     -3.75e+29     -4.03  -4.0266
*   -0.625        648.42     -8.52275   -2.5e+29 - -5.875       -2.5e+29     -2.65  -2.6478
   -0.5625       677.128     -5.33085  -1.25e+29 - -3.9375     -1.25e+29     -1.39  -1.3933
*     -0.5       707.107     -2.71467         -5 - -2              -2.29    -0.715  -0.71467
   -0.4375       738.413     -1.11381      -3.15 - 0.3             -2.04     -1.41  -1.4138
*   -0.375       771.105    -0.435544       -1.3 - 0.3            -0.864    -0.736  -0.73554
   -0.3125       805.245    -0.219041      -0.95 - 0.3            -0.731    -0.519  -0.51904
*    -0.25       840.896    -0.141133       -0.6 - 0.3            -0.459    -0.441  -0.44113
   -0.1875       878.126   -0.0875041       -0.5 - 0.3            -0.412    -0.388  -0.3875
*   -0.125       917.004   -0.0410953       -0.4 - 0.3            -0.359    -0.341  -0.3411
   -0.0625       957.603   -0.0104537      -0.35 - 0.3             -0.34     -0.31  -0.31045
*        0          1000    -0.000186       -0.3 - 0.3              -0.3      -0.3  -0.29981
    0.0625       1044.27  -0.00876107      -0.35 - 0.3            -0.341    -0.309  -0.30876
*    0.125       1090.51   -0.0305468       -0.4 - 0.3            -0.369    -0.331  -0.33055
    0.1875       1138.79   -0.0631295       -0.5 - 0.3            -0.437    -0.363  -0.36313
*     0.25       1189.21    -0.121744       -0.6 - 0.3            -0.478    -0.422  -0.42174
    0.3125       1241.86     -0.27963      -0.95 - 0.3             -0.67     -0.58  -0.57963
*    0.375       1296.84    -0.739691       -1.3 - 0.3             -0.56     -1.04  -0.56031
    0.4375       1354.26      -1.8541      -3.15 - 0.3              -1.3     -2.15  -1.2959
*      0.5       1414.21     -3.88113         -5 - -2              -1.12     -1.88  -1.1189
    0.5625       1476.83     -6.65429  -1.25e+29 - -3.9375     -1.25e+29     -2.72  -2.7168
```

```
*    0.625      1542.21     -9.76734   -2.5e+29 - -5.875      -2.5e+29      -3.89  -3.8923
    0.6875      1610.49     -12.9204  -3.75e+29 - -7.8125    -3.75e+29      -5.11  -5.1079
*     0.75      1681.79     -15.9786     -5e+29 - -9.75         -5e+29      -6.23  -6.2286
    0.8125      1756.25     -18.8987  -6.25e+29 - -11.6875   -6.25e+29      -7.21  -7.2112
*    0.875      1834.01     -21.6758   -7.5e+29 - -13.625     -7.5e+29      -8.05  -8.0508
    0.9375      1915.21     -24.3198  -8.75e+29 - -15.5625   -8.75e+29      -8.76  -8.7573
*        1         2000     -26.8437     -1e+30 - -17.5         -1e+30      -9.34  -9.3437
    1.0625      2088.55     -29.2608     -1e+30 - -19.0312      -1e+30      -10.2  -10.23
*    1.125      2181.02     -31.5837     -1e+30 - -20.5625      -1e+30        -11  -11.021
    1.1875      2277.58     -33.8234     -1e+30 - -22.0938      -1e+30      -11.7  -11.73
*     1.25      2378.41     -35.9891     -1e+30 - -23.625       -1e+30      -12.4  -12.364
    1.3125      2483.72     -38.0893     -1e+30 - -25.1562      -1e+30      -12.9  -12.933
*    1.375      2593.68     -40.1316     -1e+30 - -26.6875      -1e+30      -13.4  -13.444
    1.4375      2708.51     -42.1217     -1e+30 - -28.2188      -1e+30      -13.9  -13.903
*      1.5      2828.43     -44.0655     -1e+30 - -29.75        -1e+30      -14.3  -14.316
    1.5625      2953.65     -45.9672     -1e+30 - -31.2812      -1e+30      -14.7  -14.686
*    1.625      3084.42     -47.8311     -1e+30 - -32.8125      -1e+30        -15  -15.019
    1.6875      3220.98     -49.6622     -1e+30 - -34.3438      -1e+30      -15.3  -15.318
*     1.75      3363.59      -51.462     -1e+30 - -35.875       -1e+30      -15.6  -15.587
    1.8125       3512.5     -53.2348     -1e+30 - -37.4062      -1e+30      -15.8  -15.829
*    1.875      3668.02     -54.9819     -1e+30 - -38.9375      -1e+30        -16  -16.044
    1.9375      3830.41     -56.7075     -1e+30 - -40.4688      -1e+30      -16.2  -16.239
*        2         4000     -58.4104     -1e+30 - -42           -1e+30      -16.4  -16.41
    2.0625       4177.1     -60.0953     -1e+30 - -43.1875      -1e+30      -16.9  -16.908
*    2.125      4362.03     -61.7638     -1e+30 - -44.375       -1e+30      -17.4  -17.389
    2.1875      4555.15     -63.4158     -1e+30 - -45.5625      -1e+30      -17.9  -17.853
*     2.25      4756.83     -65.0527     -1e+30 - -46.75        -1e+30      -18.3  -18.303
    2.3125      4967.43     -66.6749     -1e+30 - -47.9375      -1e+30      -18.7  -18.737
*    2.375      5187.36     -68.2832     -1e+30 - -49.125       -1e+30      -19.2  -19.158
    2.4375      5417.02     -69.8823     -1e+30 - -50.3125      -1e+30      -19.6  -19.57
*      2.5      5656.85     -71.4687     -1e+30 - -51.5         -1e+30        -20  -19.969
    2.5625       5907.3      -73.038     -1e+30 - -52.6875      -1e+30      -20.4  -20.35
*    2.625      6168.84     -74.5995     -1e+30 - -53.875       -1e+30      -20.7  -20.724
    2.6875      6441.96     -76.1484     -1e+30 - -55.0625      -1e+30      -21.1  -21.086
*     2.75      6727.17     -77.6757     -1e+30 - -56.25        -1e+30      -21.4  -21.426
    2.8125      7025.01     -79.1847     -1e+30 - -57.4375      -1e+30      -21.7  -21.747
*    2.875      7336.03     -80.6729     -1e+30 - -58.625       -1e+30        -22  -22.048
    2.9375      7660.83     -82.1267     -1e+30 - -59.8125      -1e+30      -22.3  -22.314
*        3         8000     -83.5288     -1e+30 - -61           -1e+30      -22.5  -22.529
    3.0625      8354.19     -84.9032     -1e+30 - -61.5625      -1e+30      -23.3  -23.341
*    3.125      8724.06     -86.1873     -1e+30 - -62.125       -1e+30      -24.1  -24.062
    3.1875      9110.31     -87.3833     -1e+30 - -62.6875      -1e+30      -24.7  -24.696
*     3.25      9513.66     -88.4815     -1e+30 - -63.25        -1e+30      -25.2  -25.232
    3.3125      9934.86     -89.4638     -1e+30 - -63.8125      -1e+30      -25.7  -25.651
*    3.375      10374.7     -90.2921     -1e+30 - -64.375       -1e+30      -25.9  -25.917
    3.4375        10834     -90.9816     -1e+30 - -64.9375      -1e+30        -26  -26.044
*      3.5      11313.7     -91.5181     -1e+30 - -65.5         -1e+30        -26  -26.018
    3.5625      11814.6     -91.9365     -1e+30 - -66.0625      -1e+30      -25.9  -25.874
*    3.625      12337.7      -92.253     -1e+30 - -66.625       -1e+30      -25.6  -25.628
    3.6875      12883.9     -92.4706     -1e+30 - -67.1875      -1e+30      -25.3  -25.283
*     3.75      13454.3     -92.6296     -1e+30 - -67.75        -1e+30      -24.9  -24.88
    3.8125        14050     -92.7324     -1e+30 - -68.3125      -1e+30      -24.4  -24.42
*    3.875      14672.1     -92.8065     -1e+30 - -68.875       -1e+30      -23.9  -23.931
    3.9375      15321.7     -92.8559     -1e+30 - -69.4375      -1e+30      -23.4  -23.418
*        4        16000     -92.8977     -1e+30 - -70           -1e+30      -22.9  -22.898
    4.0625      16708.4     -92.9397     -1e+30 - -70           -1e+30      -22.9  -22.94

Correction: -19.987 db, amp. factor 0.100149

Maximum relative error in decibels: -0.299814




Plotting from 1 to 130, range -108.614 to 1.67866.

X                           #                                   +
 X                          #                                   +
  X                         #                                   +
   X                         #                                  +
    X                        #                                  +
     X                        #                                 +
      *X                      #                                 +
        X                     #                                 +
```

```
         X                     #                               +
          X                    #                               +
           X                    #                              +
            X                   #                              +
             X                  #                              +
              X                  #                             +
               X                 #                             +
                X                 #                            +
17               *X               #                            +
18                 X               #                           +
19                  X               #                          +
20                   X               #                         +
21                    X               #                        +
22                     X              #                        +
23                      X              #                       +
24                       X              #                      +
25                        *X             #                     +
26                          X             #                    +
27                           X             #                   +
28                            X            #                   +
29                             X            #                  +
(30,-63.3719)                   X            #                 +
(31,-61.7059)                    *X           #                +
(32,-60.0241)                      X           #               +
(33,-58.3254)                       X           #              +
(34,-56.6078)                        X           #             +
(35,-54.8694)                         X           #            +
(36,-53.1081)                          *X          #           +
(37,-51.3215)                            X          #          +
(38,-49.5062)                             X          #         +
(39,-47.6599)                              *X         #        +
(40,-45.7781)                                X         #       +
(41,-43.8576)                                 X         #      +
(42,-41.8933)                                  *X        ##    +
(43,-39.8802)                                    X         #   +
(44,-37.8118)                                     *X        #  +
(45,-35.6812)                                       X        # +
(46,-33.4799)                                        *X       #+
(47,-31.1987)                                          X       +
(48,-28.8256)                                           *X     +#
(49,-26.3476)                                             *X   + #
(50,-23.7494)                                               *X + ##
(51,-21.0133)                                                 *X    #
(52,-18.1217)                                                  +*X   #
(53,-15.0614)                                                  +  *X  ##
(54,-11.8391)                                                  +    *X  #
(55,-8.52275)                                                  +       *X##
(56,-5.33085)                                                  +++++     *X#
(57,-2.71467)                                                        ++++++*X
(58,-1.11381)                                                              ++X#
(59,-0.435544)                                                               X#
(60,-0.219041)                                                               +X
(61,-0.141133)                                                               +X
(62,-0.0875041)                                                              +X
(63,-0.0410953)                                                               X
(64,-0.0104537)                                                               X
(65,-0.000186)                                                                X
(66,-0.00876107)                                                              X
(67,-0.0305468)                                                               X
(68,-0.0631295)                                                              +X
(69,-0.121744)                                                               +X
(70,-0.27963)                                                                +X
(71,-0.739691)                                                               X#
(72,-1.8541)                                                                X #
(73,-3.88113)                                                         +++++X##
(74,-6.65429)                                                  ++++++    X*#
(75,-9.76734)                                                  +       X* #
(76,-12.9204)                                                  +     X* ##
(77,-15.9786)                                                  +  X*   #
(78,-18.8987)                                                  +X*   ##
(79,-21.6758)                                                 X*    #
(80,-24.3198)                                               X* +   #
(81,-26.8437)                                              X   + ##
(82,-29.2608)                                            X*    +#
(83,-31.5837)                                          X*      +
(84,-33.8234)                                         X       #+
(85,-35.9891)                                       X*       # +
(86,-38.0893)                                      X        #  +
```

```
(87,-40.1316)                                    X*        #   +
(88,-42.1217)                                   X         #    +
(89,-44.0655)                                 X*        ##     +
(90,-45.9672)                                X         #       +
(91,-47.8311)                               X         #        +
(92,-49.6622)                             X*         #         +
(93,-51.462)                             X          #          +
(94,-53.2348)                           X          #           +
(95,-54.9819)                         X*          #            +
(96,-56.7075)                        X           #             +
(97,-58.4104)                       X           #              +
(98,-60.0953)                      X           #               +
(99,-61.7638)                     X           #                +
(100,-63.4158)                  X*           #                 +
101                            X            #                  +
102                           X            #                   +
103                          X             #                   +
104                         X             #                    +
105                        X             #                     +
106                      X*             #                      +
107                     X              #                       +
108                    X              #                        +
109                   X               #                        +
110                  X               #                         +
111                 X               #                          +
112                X               #                           +
113               X               #                            +
114              X                #                            +
                X                #                             +
               X                 #                             +
              X                 #                              +
              X                 #                              +
             X                  #                              +
             X                 #                               +
            X                  #                               +
            X                 #                                +
            X                 #                                +
            X                 #                                +
           X                 #                                 +
           X                 #                                 +
           X                #                                  +
           X                #                                  +
           X                #                                  +
           X                #                                  +
Plotting from 1 to 130, range -108.614 to 1.67866.
```

3.5 Realisation of Digital One-third-octave-band Filters

The following implementation of one-third-octave-band filters is proposed:

The sampled signal is subsequently filtered by three mirror resonance filters defined by (3.0, then by three resonance filters defined by (3.0 then by two low pass filters defined by equations (2.0, (2.0 and (2.0, and finally multiplied by the corrective factor.

The calculated data for all one-third-octave-band filters is listed in Table 3.6. The data is valid for sampling rate of $10^5 s^{-1}$. For each filter the central frequency f_{cent} is specified first, designated *"fcent"*. Three pairs of parameters for mirror resonance filters and three pairs for resonance filters follow, and finally the corrective factor is stated, designated *"amp. factor"*.

For mirror resonance and resonance filters parameters designated *r1, r2, r3, r6, r7* and *r8* determine the characteristic frequencies (f_0 in (3.0 or (3.0) of the corresponding filters. Actual characteristic frequencies are obtained by multiplying the central frequency of the octave-band filter by 2 raised to the appropriate parameter, namely

$$f_0^{(i)} = f_{cent} \cdot 2^{r_i}, \tag{3.86}$$

r_i being the parameter designated *r1, r2,* etc. Parameters designated *h1, h2, h3, h6, h7* and *h8* are correspond to the parameter h_r of each individual resonance or mirror resonance filter, as defined by (3.0 or (3.0. Coefficients for each individual resonance or mirror resonance filter are calculated from r_i and h_i, which are listed in Table 3.6, by using (3.0 and (3.0, and according to (3.0 for mirror resonance filters or according to (3.0 for resonance filters. Note again that three pairs of parameters for mirror resonance filters specified by (3.0 are listed first and then three pairs of parameters for resonance filters specified by (3.0 follow.

Parameters for the two digital low pass filters incorporated in each one-third-octave-band filter are not listed in the table. For each of these filters the characteristic frequency from equation (2.0 is one octave higher than the midband frequency of the corresponding one-third-octave-band filter, i.e.

$$f_0 = 2\, f_{cent}\,. \tag{3.87}$$

For each filter the error measure *maxerr* as is defined in (3.0 is stated in Table 3.6, designated *"Maximum relative error in decibels"*. Permeability at f_{cent} is also stated in the table and is designated *"Shift"*.

In all cases the same parameters refer to several (up to three) adjacent filters. In these cases all appropriate mid-band frequencies (designated *fcent*) are stated, separated by commas, and the error measure refers to the filter whose central frequency coincides with one of the octave-band filters. Central frequencies are stated as nominal frequencies, i.e. rounded to a few digits, while the corresponding exact frequencies can be calculated from (3.0.

Table 3.6: parameters for one-third-octave-band filters.

```
fcent: 25, 31.5, 40

r1: -0.324298
h1: 0.145933

r2: -0.184024
h2: 0.159291

r3: -0.0930019
h3: 0.186468

r6: 0.0518724
h6: 0.194747

r7: 0.1643
h7: 0.132904

r8: 0.263865
h8: 0.131516


TOTAL correction: -69.6281 db, amp. factor 0.000330063

Shift: -0.298058 db, amp. factor 0.966267

Maximum relative error in decibels: -0.298058
```

```
=====================================


fcent: 50, 62.5, 80

r1: -0.337524
h1: 0.149728

r2: -0.166002
h2: 0.145054

r3: -0.100627
h3: 0.184257

r6: 0.0552794
h6: 0.192044

r7: 0.148092
h7: 0.132872
```

```
r8: 0.269085
h8: 0.112353


TOTAL correction: -70.6526 db, amp. factor 0.000293339

Shift: -0.299821 db, amp. factor 0.966071

Maximum relative error in decibels: -0.299821



=====================================



fcent: 100, 125, 160

r1: -0.338896
h1: 0.144917

r2: -0.164465
h2: 0.149383

r3: -0.101887
h3: 0.185517

r6: 0.0547839
h6: 0.193228

r7: 0.148105
h7: 0.132927

r8: 0.271272
h8: 0.111169


TOTAL correction: -70.4729 db, amp. factor 0.000299471

Shift: -0.300237 db, amp. factor 0.966024

Maximum relative error in decibels: -0.299763


=====================================


fcent: 200, 250, 315

r1: -0.341585
h1: 0.139504
```

```
r2: -0.170891
h2: 0.148572

r3: -0.0980454
h3: 0.182906

r6: 0.0534306
h6: 0.188744

r7: 0.147883
h7: 0.13359

r8: 0.270757
h8: 0.106387


TOTAL correction: -70.6729 db, amp. factor 0.000292654

Shift: -0.295061 db, amp. factor 0.9666

Maximum relative error in decibels: -0.0904119


=====================================


fcent: 400, 500, 630

r1: -0.324418
h1: 0.126896

r2: -0.163779
h2: 0.136636

r3: -0.0911893
h3: 0.173532

r6: 0.0623338
h6: 0.186191

r7: 0.146582
h7: 0.123562

r8: 0.278221
h8: 0.10534


TOTAL correction: -71.9802 db, amp. factor 0.000251762

Shift: -0.303437 db, amp. factor 0.965669

Maximum relative error in decibels: -0.296563
```

```
=====================================

fcent: 800, 1000, 1250

r1: -0.320393
h1: 0.14593

r2: -0.167378
h2: 0.139848

r3: -0.0902203
h3: 0.170174

r6: 0.0603413
h6: 0.185487

r7: 0.144738
h7: 0.124178

r8: 0.283092
h8: 0.101997

TOTAL correction: -71.9513 db, amp. factor 0.000252601

Shift: -0.300095 db, amp. factor 0.96604

Maximum relative error in decibels: -0.299905

=====================================

fcent: 1600, 2000, 2500

r1: -0.325025
h1: 0.130185

r2: -0.164835
h2: 0.137968

r3: -0.0899109
h3: 0.171826

r6: 0.0612631
h6: 0.186335

r7: 0.145084
h7: 0.124646
```

```
r8: 0.277717
h8: 0.105782


TOTAL correction: -72.0772 db, amp. factor 0.000248966

Shift: -0.300937 db, amp. factor 0.965947

Maximum relative error in decibels: -0.299063



=====================================



fcent: 3150, 4000, 5000

r1: -0.322918
h1: 0.13168

r2: -0.163941
h2: 0.138304

r3: -0.0899351
h3: 0.174027

r6: 0.0604311
h6: 0.186876

r7: 0.141932
h7: 0.125432

r8: 0.280723
h8: 0.113083


=====================================


fcent: 6300, 8000, 10000

r1: -0.287541
h1: 0.128999

r2: -0.186372
h2: 0.146092

r3: -0.0889741
h3: 0.171414

r6: 0.0658958
```

```
h6: 0.199971

r7: 0.12158
h7: 0.145007

r8: 0.25016
h8: 0.123427


TOTAL correction: -73.0787 db, amp. factor 0.000221852

Shift: -0.29986 db, amp. factor 0.966066

Maximum relative error in decibels: -0.29986



=====================================


fcent: 12500, 16000, 20000

r1: -0.272164
h1: 0.104209

r2: -0.199758
h2: 0.114685

r3: -0.0897215
h3: 0.171814

r6: 0.0533778
h6: 0.183743

r7: 0.0802801
h7: 0.15337

r8: 0.310986
h8: 0.139492


TOTAL correction: -76.524 db, amp. factor 0.00014921

Shift: -0.300111 db, amp. factor 0.966039

Maximum relative error in decibels: -0.299889

=====================================
```

3.5.1 Band Pass Filters for Sampling Rate of 48 kHz

If the sampling rate will be 48 instead of 100 kHz, the coefficients stated for one octave higher central frequencies should be taken from the above tables in order to calculate filter parameters for given central frequencies *f*. This is because for a given central frequency *f* with sampling rate 48 kHz, situation would look similar at $2f$ (approximately) at the sampling rate of 100 kHz, what concerns band pass filter response.

4 COMPLIANCE WITH STANDARDS

Algorithms for implementation of frequency weighting, time weighting, etc., proposed in Chapter 2, are designed in compliance with standards [5], [6], [7] and [8]. Algorithms for implementation of octave-band and third-octave-band filters proposed in Chapter 3 are designed in compliance with the standard [9].

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